Physics Problem Solving

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Outline of Topics

- Introduction

- Research Literature on Problem Solving
  - Definitions of “problem” and “problem solving”
  - Theoretical frameworks from psychology
  - General and specific strategies
  - Expert-novice characteristics

- Problem Solving Rubric
  - Problem solving assessment strategies
  - Constraints and Considerations
  - U of MN problem solving rubric

Introduction

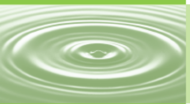
- *Problem solving* is considered a fundamental part of learning physics.
  - To help improve the teaching & learning of physics problem solving, systematic research began in the 1970's and early 1980's to understand these difficult tasks.
  - This is a primary subfield of Physics Education Research (PER)

- The capacity to solve problems is considered an essential skill for citizens of today's technological society.

- Problem solving is a skill desired by future employers, and science & engineering programs.

- Problem solving is explicitly stated as a proposed learning outcome for undergraduates at many institutions, including the University of Minnesota[1].

[1]Carney, A. (2006, October). What do we want our students to learn? *Transform*, *1*, 1-6.



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What do we want our students to learn?

By Arlene Carn

NEW LEA… OUTCOM… AT THE UNIVERS… OF MINNES…

Higher education isn't meeting the public's needs, or so say Frank Newman, Lara Couturier, and Jamie Scurry, members of Brown University's Futures Project. The Project analyzed the impact of the new competition on higher education and claimed, "Colleges have been focusing their energies on a form of competition based not on improving graduates' skills and knowledge but on institutional prestige and revenues" (p. 1). It specifically points to the college rankings in publications like *U.S. News & World Report, The Princeton Review,* and *The Financial Times.* In their critique, The Futures Project identified several key areas that must now draw higher education's attention, including the need "to elevate the status of teaching to that of research" (p. 2). The authors phrased this concern as a question: "Has the institution recognized the centrality of teaching and learning, even if it is a research university?" (p. 4).

Even though the University of Minnesota is now deeply engaged in a strategic positioning initiative whose goal is to make us one of the top three public research institutions in the world, we also need to ask

no longer an exceptional event; it's an expectation that marks a change in institutional culture. It's a true cultural change. In part, this shift is evidenced by the advent of several central units that have emerged during the past fifteen years, such as the Center for Teaching and Learning (1991), the Digital Media Center (1995), the Academy of Distinguished Teachers (1999), and the SMART Learning Commons (2004). With the formation of the department of Postsecondary Teaching and Learning (2006), the University deepened its commitment to dedicated scholarship in this field. The existence of all of these entities, and, more importantly, their active involvement and participation in every phase of university life as vocal spokespersons and advocates sends important signals to new generations of teachers and learners.

Another sign of institutional change is the questions we ask ourselves. We no longer feel the need to choose between excellent teaching and top-flight research; we now ask how we can best balance innovative research with effective teaching. We want to investigate how the two critical functions work together in our lives as faculty.

We also have to ask ourselves the hard questions: Can really exceptional teaching exist without true learning? Do we have superb

of Higher Education, Ann E. Austin wrote, "For at least the past decade, the public at large and government representatives in particular have expressed growing skepticism about the work carried out in the academy" (p. 121). This skepticism has led to a greater demand for assessment and accountability, so CESL began to prepare for the accreditation visit by focusing on these issues and in doing so made a surprising discovery: there was no university-wide statement declaring what the University gives its students.

To move us toward a common set of student outcomes, we decided to approach various individuals and groups, asking them one question: "What should graduates with a bachelor's degree be able to do when they leave the university?" We spent a year developing these learning outcomes and another year vetting them both on campus and across the country. We got feedback from the Academy of Distinguished Teachers, the Senate Committee on Educational Policy, the Assessment Conference at IUPUI, the American Association for Higher Education, and eventually from the Higher Learning Commission during our accreditation site visit. Indeed, the site committee looked very positively on these outcomes.

We proposed the following. At the time of receiving a bachelor's

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and social sciences

7. **skills for effective citizenship and life-long learning.**

If one combines these seven learning outcomes with their elaboration (see inset), one sees that CESL's interest in student learning focuses initially on disciplinary knowledge but gradually expands to include "public" knowledge; it hopes to facilitate the student's learning while at the University but also encourage his or her lifelong education. Vartan Gregorian, former president of Brown University and the New York Public Library, worried that higher education "has largely come to serve as a job-readiness program," and the Futures Project echoed this concern when it asked, "Has [higher education] recognized that education includes more than simply job skills, that it entails development and practice of civic skills?" (pp. 1, 4). I believe these outcomes demonstrate that the University of Minnesota hopes to achieve far more than job training. We strive to prepare students to be effective citizens and life-long learners who display intellectual curiosity; an ability to communicate, synthesize, and evaluate complex information; and an openness to individuals with diverse perspectives and backgrounds.

(continued on page 6)

The ability to identify, define, and solve problems

What do we want our students to learn? *(continued)*

Proposed Undergraduate Learning Outcomes

Foundational Life-Long Learning and Citizenship Goals	Elaboration/Examples
At the time of receiving a bachelor's degree, students will demonstrate:	University of Minnesota graduates:
1. the ability to identify, define, and solve problems	• recognize the complexity and ambiguity inherent in many problems • can evaluate and synthesize knowledge and frame logical arguments based on this knowledge • understand and use the scientific method and other modes of problem solving
2. the ability to locate and evaluate information	• can access information as needed and work effectively with modern information technologies • understand and practice the responsible and ethical use of information
3. mastery of a body of knowledge and mode of inquiry	• know the facts, theories, and concepts central to their discipline • display appropriate disciplinary literacy and sophistication • understand the relationships between the methods and content of their discipline • understand the social and ethical context and implications of disciplinary knowledge and endeavors
4. an understanding of diverse philosophies and cultures in a global society	• understand the philosophical, artistic, scientific, and political roots of civilization • are able to put issues in their historical, philosophical, and societal context • can work with individuals from diverse backgrounds, perspectives, and disciplines
5. the ability to communicate effectively	• communicate ideas and information effectively in appropriate formats to different audiences and in different contexts • engage in constructive discussion by listening accurately, understanding the perspectives of others, and demonstrating civility and respect
6. an understanding of the role of creativity, innovation, discovery, and expression in the arts and humanities and in the natural and social sciences	• possess a sufficient foundational knowledge to understand applications and impacts of art, humanities, and science on daily life • can make aesthetic and logical judgments • understand connections between disciplines
7. skills for effective citizenship and life-long learning (see "Regents" in References)	• display intellectual curiosity, flexibility, and openness • are able to reflect upon and articulate their own values • understand and practice professional and ethical behavior • are aware of personal strengths and weaknesses and are prepared for life after college • understand the nature and importance of responsible citizenship

OK, so…

- Problem solving skill is valued by society, institutions of higher learning, and physics courses….but how do we measure progress toward that goal?

- *How is problem solving skill assessed?*

> In a recent NRC report Augustine (2006) states:
>
> The No Child Left Behind legislation requires testing of students' knowledge of science beginning in 2006-2007, and the science portion of the NAEP is being redesigned. Development of such assessments raises profound methodologic issues such as *how to assess inquiry and problem-solving skills* using traditional large scale testing formats (p. 264)

- Answering this question requires a definition of what problem solving is, and what problem solving looks like.

Augustine, N. (2006). *Rising Above The Gathering Storm: Energizing and Employing America for a Brighter Economic Future.* Washington, DC: National Academies Press.

What is a “problem”?
What is “problem solving”?

- Newell & Simon (1972) write that “A person is confronted with a *problem* when he wants something and does not know immediately what series of actions he can perform to get it” (p. 72).

- Martinez (1998) states, “problem solving is the process of moving toward a goal when the path to that goal is uncertain” (p. 605)

- What is a “problem” for one person might not be a problem for another person.

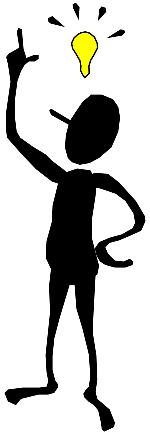
- If the steps to reach a solution are immediately known, this is an “exercise” for the solver.

Martinez, M. E. (1998). What is Problem Solving? *Phi Delta Kappan*, *79*, 605-609.

Newell, A., & Simon, H. A. (1972). *Human problem solving*. Englewood Cliffs, NJ: Prentice Hall.

Theoretical Frameworks from Psychology

- <u>Early theories (behaviorist)</u>
  - Observations of animals in puzzle boxes: some behavior “trial and error”
  - Conditioning based on stimulus-response associations (response hierarchy)



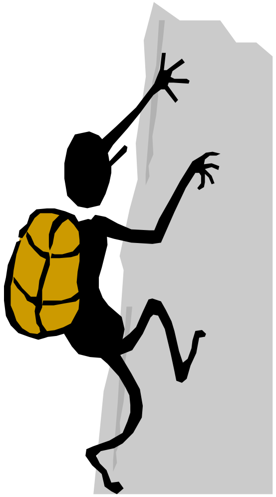
- <u>Cognitive perspective</u>
  - Gestalt theory – problem solving is a process of mental restructuring until a point of “insight”
  - Information processing theory:
    - Short-term or “working” memory; limited in capacity
    - Long-term memory; info is encoded, to be retrieved it must be activated (brought into working memory)
    - Terms: *metacognition*, *cognitive load*

General PS Strategies

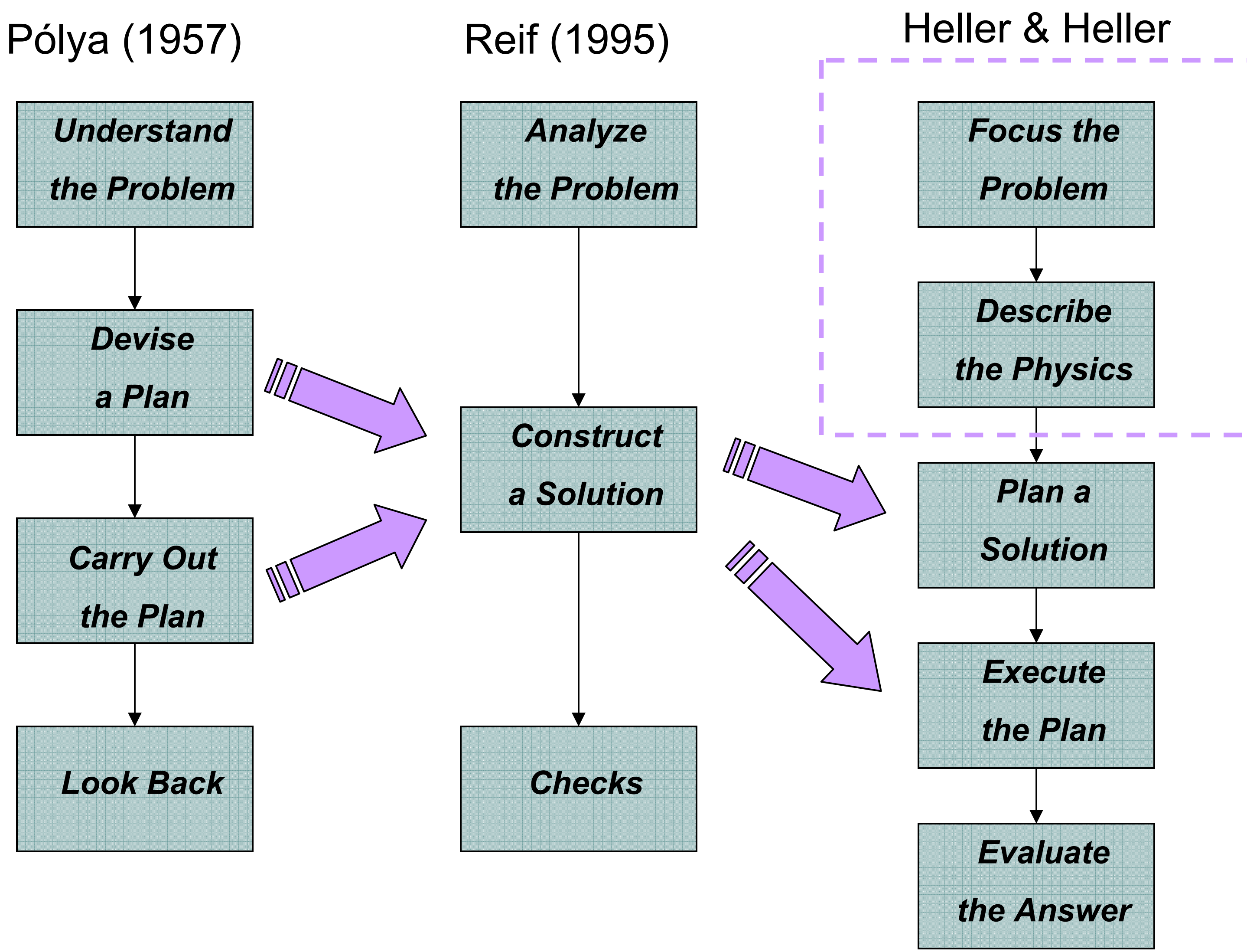
- *Algorithms* are step-by-step procedures that will guarantee a solution every time, if used correctly (like tying your shoe).

- *Heuristics* are general strategies or "rules of thumb" for solving problems.

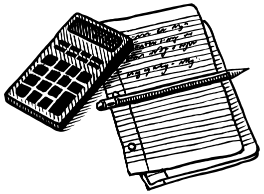
- Heuristics include:
  - Combining algorithms
  - Hill climbing
  - Successive refinements
  - Means-ends analysis
  - Working backward
  - External representations



Specific strategies



Expert-novice characteristics



- Inexperienced (novice) problem solvers:
  - Little representation (jump quickly to equations)
  - haphazard formula-seeking and solution pattern matching
  - Inefficient knowledge storage in memory (must access principles individually) → high cognitive load
  - Categorize problems based on surface features or objects

- Experienced (expert) problem solvers:
  - Low-detail overview of the problem before equations (Larkin calls this a *qualitative analysis*)
  - "Chunk" information in memory as principles that can be usefully applied together → lower cognitive load (automatic processes)
  - Categorize problems based on deep structure (principles)
  - Check their solution based on expectations for extreme values or special cases of parameters in the problem

Chi, M. T., Feltovich, P. J., & Glaser, R. (1980). Categorization and Representation of Physics Problems by Experts and Novices. *Cognitive Science*, *5*, 121-152.

Larkin, J. H. (1979). Processing information for effective problem solving. *Engineering Education*, *70*(3), 285-288.

Problem solving assessment

- In most Physics classes:
  - Students’ problem solutions on homework or exams are given a score based on the correctness of the algebraic or numerical solution.
  - Partial credit is given for particular characteristics of the written solution, as compared to an ideal solution developed by the instructor.
  - HOWEVER: This “score” does not adequately describe a student’s problem solving performance.
  - A different kind of instrument is required to determine the nature of a student’s approach to the problem and assess their “expertise”.

Constraints / considerations

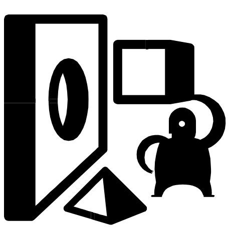
- An evaluative tool to assess problem solving performance must consider its intended use for both *research* and *instruction*.

For research:

- A common ranking scale (rubric) can facilitate comparison of student performance, in different courses or even between institutions.
- The criteria should not be biased for or against a particular problem solving strategy.
- Criteria must be described in enough detail that use of the tool can be easily learned
- Scores by different people must be reasonably consistent

For instruction:

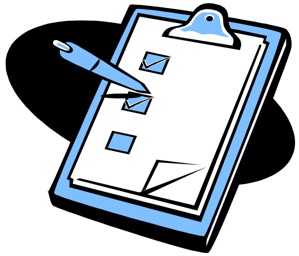
- Must be general enough that it applies to a range of problem topics.
- Criteria must be consistent with instructors' values with respect to "good" or "bad" problem solutions.



U of MN rubric

- Work began on a rubric with investigations by Heller, Keith, & Anderson (1992) and the doctoral dissertations of Jennifer Blue (1997) and Tom Foster (2000).
- Revision process has continued sporadically since then: multiple raters code example student solutions from final exams and discuss their scores.
- Rubric is based on characteristics of expert and novice problem solutions, and condenses earlier schemes into four dimensions:

1. **Physics Approach**
2. **Translation of the Physics Approach**
3. **Appropriate Mathematics**
4. **Logical Progression**



Blue, J. M. (1997). *Sex differences in physics learning and evaluations in an introductory course.* Unpublished doctoral dissertation, University of Minnesota, Twin Cities.

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Heller, P., Keith, R., & Anderson, S. (1992). Teaching problem solving through cooperative grouping. Part 1: Group versus individual problem solving. *American Journal of Physics*, *60*(7), 627-636.

Physics Approach

(from problem statement to physics description)

•*Assesses an initial conceptual understanding of the problem statement. Typically this is indicated through an explicit description of the physics principles and visual representation of the problem situation.*

0	**Nothing written can be interpreted as a physics approach.**
1	**All physics used is incorrect or inappropriate. Correct solution is not possible.**
2	**Use of a few appropriate physics principles is evident, but most physics is missing, incorrect, or inappropriate.**
3	**Most of the physics principles used are appropriate, but one or more principles are missing, incorrect, or inappropriate.**
4	**(Apparent) use of physics principles could facilitate successful solution; missing explicit statement of physics principles.**
5	**Explicit statement of physics principles could facilitate successful problem solution.**

Translation of Physics Approach

(from Physics Approach to specific equations)

•*Assesses a student's success in converting stated physics principles and situation representations into symbolic form as specific physics equations*

0	**Nothing written can be interpreted as a translation of the specified physics approach.**
1	**Fundamental error(s) in physics, such as treating vectors as scalars or not distinguishing between energy and momentum.**
2	**Missing an explicit symbol for a physics quantity or an explicit relationship among quantities essential to the specified physics approach.**
3	**Misunderstanding the meaning of a symbol for a physics quantity or the relationship among quantities.**
4	**Limited translation errors (i.e. sign error, wrong value for a quantity, or incorrect extraction of a vector component).**
5	**Appropriate translation with a correct but not explicitly defined coordinate system (such as x-y coordinate axes, current direction in a circuit, or clockwise/counterclockwise rotation).**
6	**Appropriate translation with explicitly defined coordinate system.**

Appropriate Mathematics

(from specific equations to numerical answer)

• *Assesses the performance of mathematical operations on specific physics equations to isolate the desired quantities and obtain a numerical answer.*

0	**Nothing written can be interpreted as mathematics.**
1	**Mathematics made significantly easier (than a correct solution) by inappropriate translation from problem statement to specific equations.**
2	**Solution violates rules of algebra, calculus, or geometry. "Math-magic" or other unjustified relationship produces an answer (with "reasonable" units, sign, or magnitude).**
3	**Careless math or calculation error or unreasonable answer or answer with unknown quantities**
4	**Mathematics leads from specific equations to a reasonable answer, but features early substitution of non-zero numerical values for quantities.**
5	**Appropriate mathematics with possible minor errors (i.e. sign error or calculator error) lead from specific equations to a reasonable answer with numerical values substituted in the last step.**

Logical Progression *(entire problem solution)*

•*Assesses the overall cohesiveness of a problem solution; how organized and logical the solution is. Novices often make illogical jumps or leave their solutions unfinished.*

0	**Nothing written can be interpreted as logical progression.**
1	**Haphazard solution with obvious logical breaks.**
2	**Part of the solution contradicts stated principles, the constraints shown in the explicit statement of the problem situation, or the assumptions made in another part of solution.**
3	**Solution is logical but does not achieve the target quantity or haphazard but converges to the target.**
4	**Solution is organized but contains some logical breaks.**
5	**Progress from problem statement to an answer includes extraneous steps.**
6	**Consistent progress from problem statement to answer.**

Challenges

- The rubric is still in a developmental phase
  - Further testing by multiple coders should continue with example student solutions (from a variety of problem *types* and *topics*)
- Problem solving is a complex process, but the rubric needs to be relatively simple. Where is the appropriate balance?
- Should there be the same number of criteria for each dimension? Should one dimension be weighted more than another?

  Is a “score” meaningful?
- Is “Logical Progression” two distinct characteristics?
- The language of the criteria must be easily understood by different people (pilot test). Check for inter-rater and intra-rater reliability
- The rubric should be examined for consistency with physics instructors’ values for problem solving

THERE IS STILL A LOT OF WORK TO BE DONE!!

References

- Augustine, N. (2006). *Rising Above The Gathering Storm: Energizing and Employing America for a Brighter Economic Future.* Washington, DC: National Academies Press.
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Additional Slides

Heller, Keith, & Anderson (1992)

Evidence of conceptual understanding

Does the physics description reveal a clear understanding of physics concepts and relations?
For instance:

- does the description indicate curvilinear trajectories for projectiles?
- does the solution employ unbalanced forces for an accelerating object?

Usefulness of description

Is the essential information needed for a solution present?
For example:

- do the force diagrams include all relevant forces?
- are momentum vectors both before and after an interaction clearly indicated?

Match of equations with description

Are the specific equations used consistent with the physics described?
For example:

- are vector equations used to relate vector quantities?
- are forces from description appropriately included in specific force equations?

Reasonable plan

- Does the solution indicate that sufficient equations were assembled before the algebraic manipulations of equations was undertaken?
- Does the solution include an indication of how to combine equations to obtain an answer?

Logical progression

- Does the mathematical solution progress logically from general principles to a problem-specific formulation using defined variables?
- Are numbers substituted for variables only after an algebraic solution was obtained for the unknown variable?

Appropriate mathematics

- Aside from minor mistakes, is the mathematics used reasonable?
- Or does the solution employ invalid mathematical claims in order to obtain an answer?

Jennifer Blue (1997) – 4 D

1. General Approach

a	Nothing written.
b	Invalid or inappropriate principles (general formulas) are used.
c	The solution indicates a clear misunderstanding of how the central principle(s) are systematically applied in general to physical events.
d	The solution indicates an absurd assumption or interpretation regarding certain information needed for solution of the problem. The assumption/interpretation contradicts the assumption/interpretation that the instructor feels it reasonable to expect from any student who has been actively enrolled in class up to that point in the course.
e	The solution approach is partially correct. The solution includes correct identification of the central principle; but another concept important to the solution is either omitted, or there is indication of a serious misunderstanding of this concept.
f	The solution approach is mostly correct but a serious error is made about certain features of the physical events.
g	The solution correctly uses all of the required principles. Errors in the solution are in the details of application to the specific problem, rather than in the general application of concepts and principles to physical events.

Jennifer Blue (1997) – 4 D

2. Specific Application of Physics

a	Nothing written.
b	Difficult to assess because the individual's use of principles is fundamentally flawed. Because it is difficult to characterize the nature of the individual's approach, it is impossible to determine whether or not the individual applied the ideas in a consistent manner.
c	Specific equations are incomplete. Not all of the equations needed for a correct solution are presented.
d	Confusion regarding resolution of vectors into components.
e	Wrong variable substitution: The specific equations exhibit an incorrect variable substitution.
f	Careless use of coordinate axes or inconsistent attention to direction of vector quantities: The specific equations exhibit inconsistencies with regard to the signs associated with variable quantities (e.g. In a problem where the v and a of an object are in the same direction, the equation assigns different signs to the v and a variables).
g	Careless substitution of given information: Incorrect given information is substituted into equation for specified variable.
h	Specific equations do not exhibit clear inconsistencies with student's general physics approach and solution seems quite complete in its identification of quantities and their relative directions.

Jennifer Blue (1997) – 4 D

3. Logical Progression

a	Nothing written.
b	Not applicable. Solution is essentially a one-step problem, i.e. individual's solution involves given information substituted into a single principle relationship.
c	Solution does not show a logical progression in the use of equations. The use of equations appears haphazard.
d	Solution is logical to a point, then one or more illogical or unnecessary jump is made. Student may not understand how to combine equations to isolate variables. In solution it may appear that earlier physics claims are abandoned in an attempt to reach an mathematical solution.
e	Solution is logical but unfinished.
f	Solution involves occasional unnecessary calculations but there is a logical progression of equations that leads to an answer.
g	Solution progresses from general principles to answer. (Solution proceeds in a straightforward manner toward solution.) Solution is successful in isolating desired unknown.

Jennifer Blue (1997) – 4 D

4. Appropriate Mathematics

a	Nothing written.
b	Solution is terminated for no apparent reason.
c	When an obstacle to mathematical solution (e.g. incorrect occurrence of -1) is encountered, either "math magic" or additional (non-justified) relationships are introduced in order to get an answer or the solution is terminated.
d	Solution violates rules of algebra, arithmetic, or calculus (e.g. x a + b = x a + x b). Students apparently does not have mastery of basic mathematical operations or of transitive, commutative, or distributive properties of numbers.
e	Mistakes from line to line, like sign changes.
f	Mathematics is complete and nearly correct, with only minor mistake such as a calculator error or neglected factor of 2.
g	Mathematics is correct.

Inter-rater reliability

- Fleiss' kappa is a variant of Cohen's kappa, a statistical measure of inter-rater reliability. Fleiss' kappa works for multiple raters giving categorical ratings to a fixed number of items; it is a measure of the degree of agreement that can be expected above chance.

$$\kappa = \frac{\bar{P} - \bar{P}_e}{1 - \bar{P}_e}$$

- *N* is the total number of subjects, *n* is the number of ratings per subject, and *k* is the number of categories into which assignments are made. The subjects are indexed by *i* = 1, 2,…N and categories are indexed by *j*=1,2,…*k*. Let n_{ij} represent the number of raters who assigned the i-th subject to the j-th category.

Calculate the proportion of all assignments which were to the j-th category:

$$p_j = \frac{1}{Nn} \sum_{i=1}^{N} n_{ij}$$

Calculate the extent to which raters agree for the i-th subject:

$$P_i = \frac{1}{n(n-1)} \sum_{j=1}^{k} n_{ij}(n_{ij} - 1)$$

Compute the mean of the P_i's:

$$\bar{P} = \frac{1}{N} \sum_{i=1}^{N} P_i$$

Compute the degree of agreement expected by chance:

$$\bar{P}_e = \frac{1}{N} \sum_{j=1}^{k} p_j^2$$

Inter-rater reliability

kappa	Interpretation
<0	No agreement
0.0-0.19	Poor agreement
0.20-0.39	Fair agreement
0.40-0.59	Moderate agreement
0.60-0.79	Substantial agreement
0.80-1.00	Almost perfect agreement