

2. Teaching Problem Solving

This chapter includes some of the materials we use to teach an organized, logical problem-solving strategy to students in our calculus-based physics course.

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This is an excerpt from the first chapter of *The Competent Problem Solver*, a booklet students read to help them learn how to solve physics problems. It contains an explanation of the general strategy experts in all fields use to solve real problems, a description of how this strategy can be used to solve physics problems, and an example solution to a standard kinematics problem using the strategy.

Flow Charts of the Strategy	26
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We use an overhead of the *first* flow chart to explain the problem solving strategy to students. We emphasize that this is a general strategy used in all disciplines.

After the problem-solving strategy has been modeled several times in lecture (2 - 3 weeks), we use the following five flow-charts to help students learn the problem-solving steps and the questions they should be asking at each step. A class set of these flow charts can be made and used during the cooperative-group problem solving sessions.

Blank Format Sheet for Solving Problems	32
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These format sheets are used in many different ways in our courses. Students can photocopy the sheets for solving their homework problems (see also *The Competent Problem Solver*). For the first three or four test, students solve all recitation and test problems on these sheets. Thereafter, an outline of the steps is included with all recitation and test problems, but students solve the problems on blank sheets of paper. All distributed and posted problem solutions for recitation test problems are worked out on these format sheets.

Grading Feedback

This chart of common student mistakes is intended to help instructors grade problems. For our test, each problem, group or individual, is worth 25 points. The points assigned to different portions of students' problem solutions change over the two-quarters of the algebra-based course, as shown in the table on the next page. During the first quarter, students have the most difficulty with the first two steps, so these steps are allocated more points. (This encourages students to analyze the problem qualitatively *before* they plug numbers in a formula.) As students become knowledgeable and comfortable with all the problem-solving steps, the allocation of points is more evenly distributed among the five steps.

	<u>First Quarter</u>	<u>Second Quarter</u>
Focus the Problem	7	5
Describe the Physics	8	6
Plan a Solution	5	6
Execute the Plan	3	6
Evaluate the Solution	<u>2</u>	<u>2</u>
Total	25	25

A Logical Problem Solving Strategy

Introduction

At one level, problem solving is just that, solving problems. Presented with a problem you try to solve it. If you have seen the problem before and you already know its solution, you can solve the problem by recall. Much of the time, however, you have never experienced this situation before (if you had, you would not call it a problem). Solving real problems involves making a logical chain of decisions which lead from an unclear situation to a solution. Solving physics problems is not very different from solving any kind of problem. In your professional life, you will encounter new and complex problems (after all, if your employers knew how to solve these problems, why would they pay you?). The skillful problem solver is able to invent good solutions for these new problem situations. But how does the skillful problem solver create a solution to a new problem? And how do you learn to be a more skillful problem solver?

A Logical Problem-Solving Strategy

The problem-solving strategy presented here is based on research done in a variety of disciplines such as physics, medical diagnosis, engineering, project design and computer programming. There are many similarities in the way experts in these disciplines solve problems. The most important result is that experts follow a *general strategy* for solving all complex problems. That is, experts solve *real* problems in several steps. Getting started is the most difficult step. In the first and most important step, you must accurately *visualize the situation, identify the actual problem, and identify information relevant to the problem*. At first you must deal primarily with the qualitative aspects of the situation. You must interpret the problem in light of your own knowledge and

experience. This enables you to decide what information is important, what information can be ignored, and what additional information may be needed, even though it was not explicitly provided. In this step drawing a useful picture of the problem situation is crucial to getting started correctly. A picture is worth a thousand words (if it is the right picture).

In the second step, you must *represent the problem* in terms of formal concepts and principles, whether these are concepts of engineering design, concepts of medicine, or concepts of physics. These formal concepts and principles use the accumulated knowledge of your field and thus enable you to simplify a complex problem to its essential parts. Frequently, your field has developed a formalized way to diagram the situation which helps show how the concepts are usually applied to a problem. Third, you must use your representation of the problem to *plan a solution*. Planning results in an outline of the logical steps required to obtain a solution. In many cases the logical steps are conveniently expressed as mathematics. Fourth, you must determine a solution by actually *executing* the logical steps outlined in your plan. Finally, you must *evaluate* how well the solution resolves the original problem.

The general strategy can be summarized in terms of five steps.:

- (1) Comprehend the problem.
- (2) Represent the problem in formal terms.
- (3) Plan a solution.
- (4) Execute the plan.
- (5) Interpret and evaluate the solution.

The strategy begins with the qualitative aspects of a problem and progresses toward the quantitative aspects of a problem. Each step uses information

gathered in the previous step to translate the problem into more quantitative terms and to clarify the decisions which you must make. These steps should make sense to you. You have probably used a similar strategy, without thinking about it, when you have solved problems before.

The Importance of Writing

Solving a problem requires that you constantly make decisions. This is very difficult to do if you must also remember many pieces of information and the relationships between those pieces of information. Soon you overload your brain which has only a small number of short term memory locations. You could forget important parts of the problem or the steps in a mathematical procedure. The chain of decisions you construct may even have logical flaws. Drawing pictures and diagrams and writing your procedures using words, symbols, and mathematics makes the paper a part of your extended memory. Your brain is then free to deal with the decision-making process. The single biggest mistake of novice problem solvers is not writing down enough in a form which is organized to be a useful aid to their memory. If you have had the experience of understanding how to solve a problem when someone shows you how but “getting lost” when you try to do a similar problem yourself, the effective use of writing could be your primary trouble.

A Physics-Specific Strategy

Each profession has its own specialized knowledge and patterns of thought. The knowledge and thought processes that you use in each of the steps will depend on the discipline in which you operate. Taking into account the specific nature of physics, we choose to label and interpret the five steps of the general problem solving strategy as follows:

1. **Focus the Problem**: In this step you develop a qualitative description of the problem. First, visualize the events described in the problem

using a sketch. Write down a simple statement of what you want to find out. Write down the physics ideas which might be useful in the problem and describe the approach you will use. When you finish this step, you should never have to refer to the problem statement again.

2. **Describe the Physics**: In this step you use your qualitative understanding of the problem to prepare for a quantitative solution. First, simplify the problem situation by describing it with a diagram in terms of simple physical objects and essential physical quantities. Restate what you want to find by naming specific mathematical quantities. Using the physics ideas assembled in step 1, write down equations which specify how these physical quantities are related according to the principles of physics or mathematics. The results of this step contains all of the relevant information so you should not need to refer to step 1 again.

3. **Plan the Solution**: In this step you translate the physics description into a set of equations which represent the problem mathematically by using the equations assembled in step 2. Each equation should have a specific goal to find a single unknown quantity in the problem. An equation thus used may involve a new unknown quantity which must be determined using another equation. In other words, solving the original problem usually involves creating and solving sub-problems. As you do the mathematical operations to isolate your unknown quantities, you create an outline of how to arrive at a solution. You will find that most of your effort will go into deciding how to construct this logical chain of equations with less effort spent on mathematical operations.

4. **Execute the Plan**: In this step you actually execute the solution you have planned. Plug in all of the known quantities into the algebraic

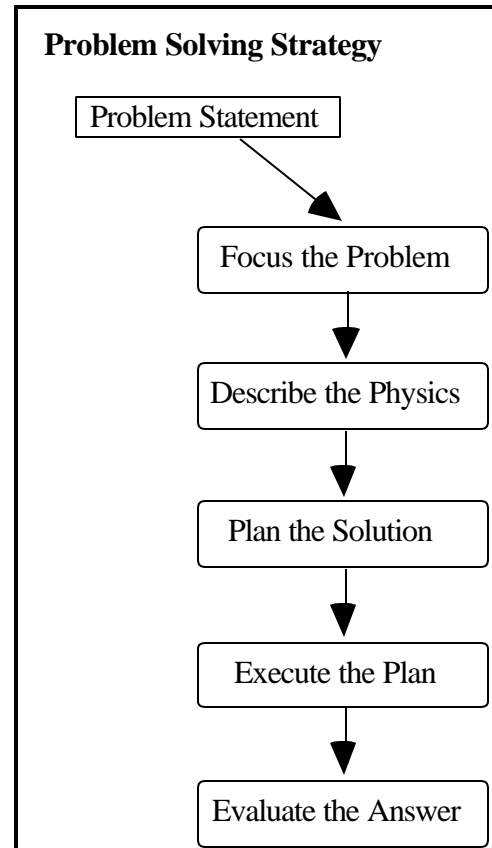
solution, which is the result of step 3, to determine a numerical value for the desired unknown quantity(ies).

5. **Evaluate the Answer:** Finally, check your work to see that it is properly stated, not unreasonable, and actually answers the question asked.

Consider each step as a translation of the previous step into a slightly different language. You begin with the full complexity of real objects interacting in the real world and through a series of decisions arrive at a simple and precise mathematical expression.

The solution to the following problem illustrates each step. On the right side of the page is the actual solution, as you might

construct it. On the left side of the page are brief descriptions of each step of the solution. We have used a familiar situation so that you can concentrate on understanding how the strategy is applied.



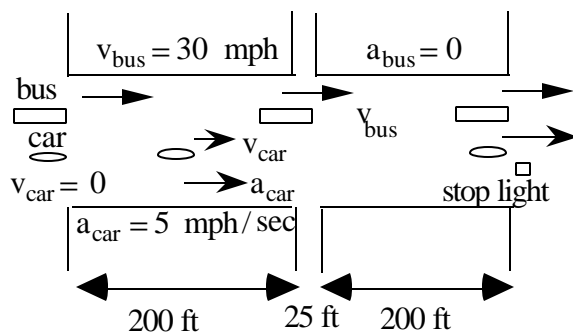
Example: Just as you turn onto the main avenue from a side street with a stop sign, a city bus going 30 mph passes you in the adjacent lane. You want to get ahead of the bus before the next stoplight which is two blocks away. Each block is 200 ft long and the side streets are 25 ft wide, while the main avenue is 60 ft wide. If you increase your speed at a rate of 5 mph each second, will you make it?

(1) **Focus the Problem** In this step of the problem solving strategy construct your initial qualitative understanding of the problem situation. Write down what you know, what you want to know, the physics you will use, and the assumptions you will make. This understanding can be usefully expressed as follows:

Picture & Given Information:

What's happening? Visualize the problem situation and make a sketch of the important objects and events.

Decide which given information may be useful and write it down on the sketch.



Question(s):

What is(are) the question(s)? Express it as some quantity to be found.

Find the distance the car travels to catch up to the bus. See if it is less than 425 feet.

Approach:

What approach shall I take? Outline the concepts which can relate the given information to the question.

Use the definition of average velocity for the bus since it travels at constant velocity.

Use the relationship between acceleration and position for the car since it travels at constant acceleration.

Initial time is when the bus and the car are first together. Final time is when the bus and the car are next together.

(2) Describe the Physics: In this step use your physics ideas to translate your initial understanding of the problem into a diagram of the actual problem. This diagram contains only idealized physical objects and representations of important physical quantities. Identify which of these physical quantities you need to find to answer the question. Write down the relationships between the quantities which will help you determine the unknowns. This information can be summarized with the following items:

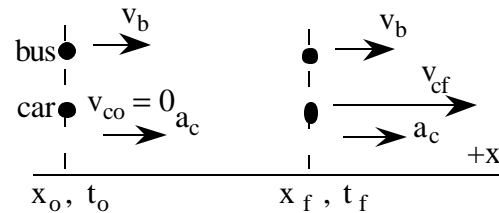
Diagram & Define Quantities:

For kinematics problems, use a motion diagram.

This diagram requires:

- * Coordinate axes.
- * Simplified representations (usually points) of objects.
- * Indication of position, velocity and acceleration of objects at important times.

Identify known and unknown quantities.



$$x_o = 0 \quad x_f = ?$$

$$t_o = 0 \quad t_f = ?$$

$$v_{co} = 0 \quad v_{cf} = ?$$

$$v_b = 30 \text{ mph} \quad a_c = 5 \text{ mph / sec}$$

Target Quantity(ies):

Decide which of your unknowns you will need to find in order answer the problem question.

$$x_f = ?$$

Quantitative Relationships :

Decide which physics principles or other mathematical relationships are applicable for the situation diagrammed above.

$$v_b = \frac{x_f - x_o}{t_f - t_o} = \frac{x_f}{t_f} \quad v_b \text{ constant}$$

$$x_f = \frac{1}{2} a_c (t_f - t_o)^2 + v_{co} (t_f - t_o) + x_o$$

$$x_f = \frac{1}{2} a_c (t_f)^2 \quad a_c \text{ constant}$$

(3) Plan the Solution: In this step translate your physics description of the problem into the particular equations, which will help you solve the problem. Always begin with an equation from your quantitative relationships containing the target quantity. If that equation contains additional unknowns, write down another equation from your quantitative relationships containing one of those unknowns. Continue until you have introduced a new equation for every unknown in your plan.

Construct Specific Equations :

Use your quantitative relationships to write specific equations relating unknown quantities to ones which are known.

Find x_f

$$x_f = \frac{1}{2} a_c (t_f)^2$$

unknowns

x_f

t_f

Find t_f

$$v_b = \frac{x_f}{t_f}$$

$$t_f = \frac{x_f}{v_b}$$

$$x_f = \frac{1}{2} a_c \left(\frac{x_f}{v_b} \right)^2$$

$$\frac{2v_b^2}{a_c} = x_f$$

Check Units:

Make sure the units on both sides of your equation are the same.

$$\frac{\left[\frac{\text{mi}}{\text{hr}} \right]^2}{\frac{\text{mi}}{\text{hr}^2}} = \text{mi} \quad \text{OK}$$

(4) Execute the Plan: In this step carry out the mathematics specified in your solution plan in order to determine a numerical value for your target quantity(ies).

Calculate Target Quantity(ies):

Put numerical values of known quantities into the equation for the target quantity. Convert units if necessary and calculate a value for the target quantity.

$$x_f = 2 \frac{\left(30 \frac{\text{mi}}{\text{hr}}\right)^2}{\left(\frac{5 \frac{\text{mi}}{\text{hr}}}{\text{s}}\right)}$$

$$x_f = 360 \left(\frac{\text{mi}}{\text{hr}}\right) \text{s}$$

$$x_f = 360 \left(\frac{\text{mi}}{\text{hr}}\right) \text{s} \left(\frac{\text{hr}}{3600 \text{s}}\right) = \frac{1}{10} \text{mi}$$

Since 0.1 miles is 528 feet, which is more than 425 feet, you do not make it.

(5) Evaluate the Answer: As a result of executing your plan, you have a numerical answer to the physics problem. In this final step, check that your answer is properly stated, not unreasonable, and complete.

Is Answer Properly Stated?:

Check that your answer has the appropriate units and sign.

Yes, miles are a correct unit for distance.

Is Answer Unreasonable?:

Check that the magnitude of your answer is not unexpectedly large or small.

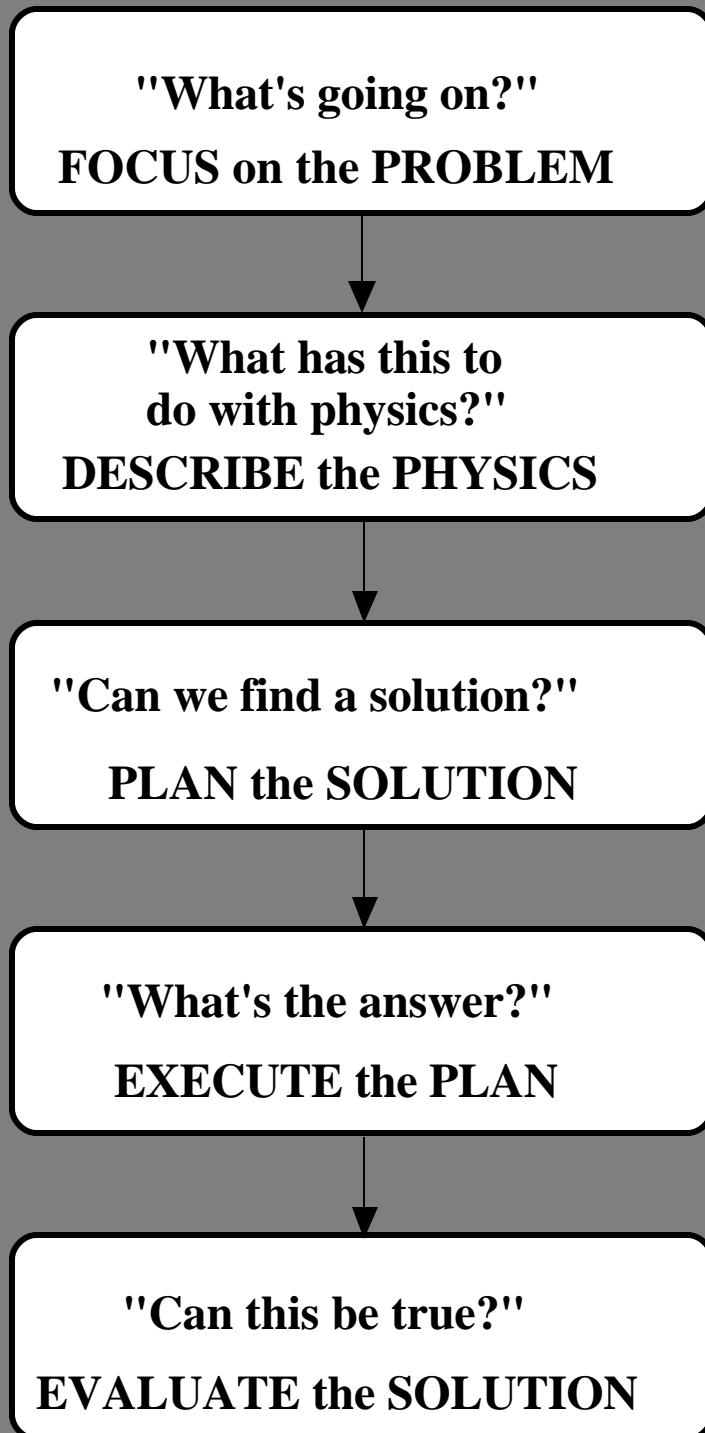
The answer is only about 100 ft longer than the 2 block distance which is not unreasonable.

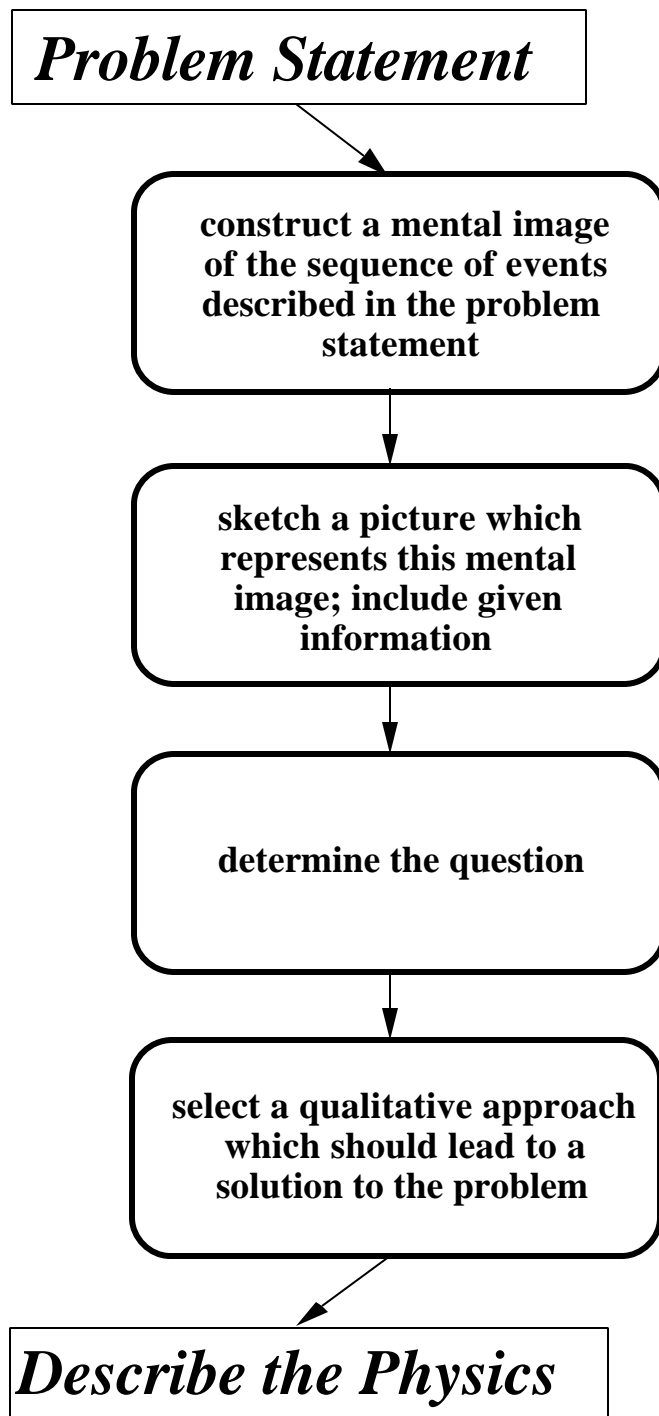
Is Answer Complete?:

Check that you have answered the original question.

The care does not make it answers the question

STEPS FOR SOLVING A PROBLEM

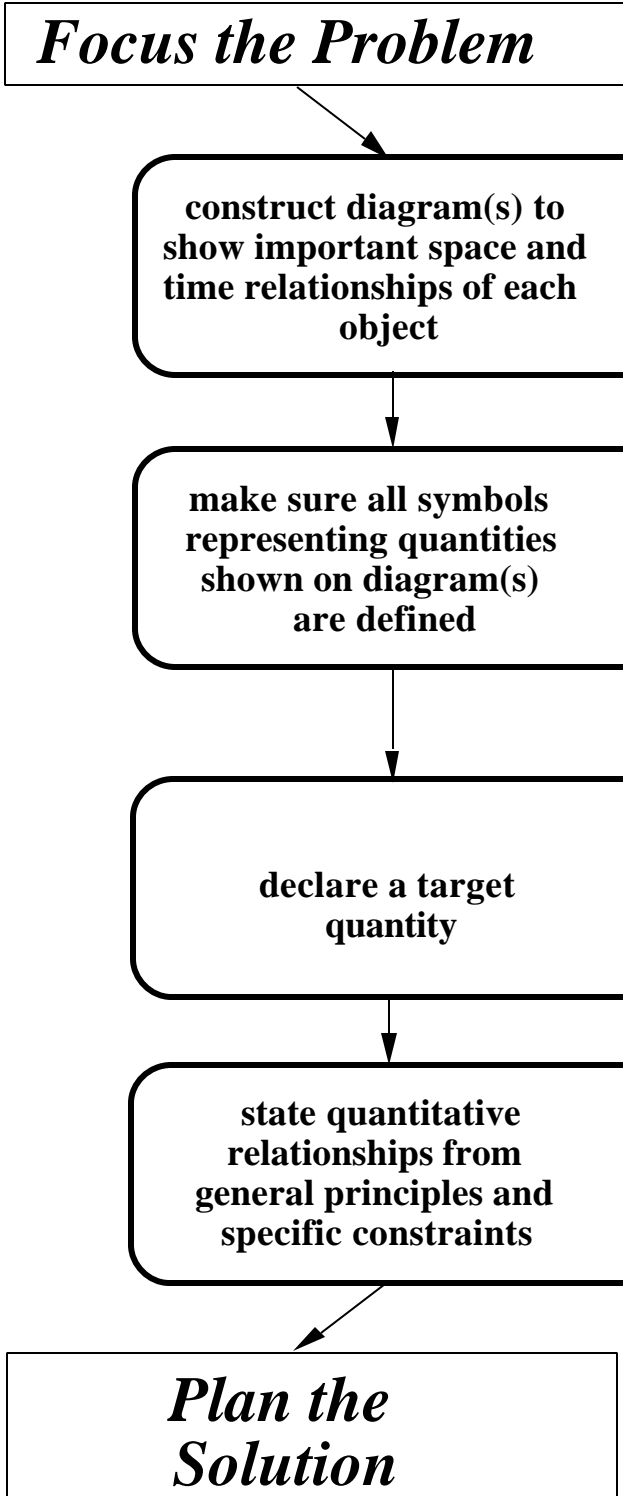




Focus the Problem

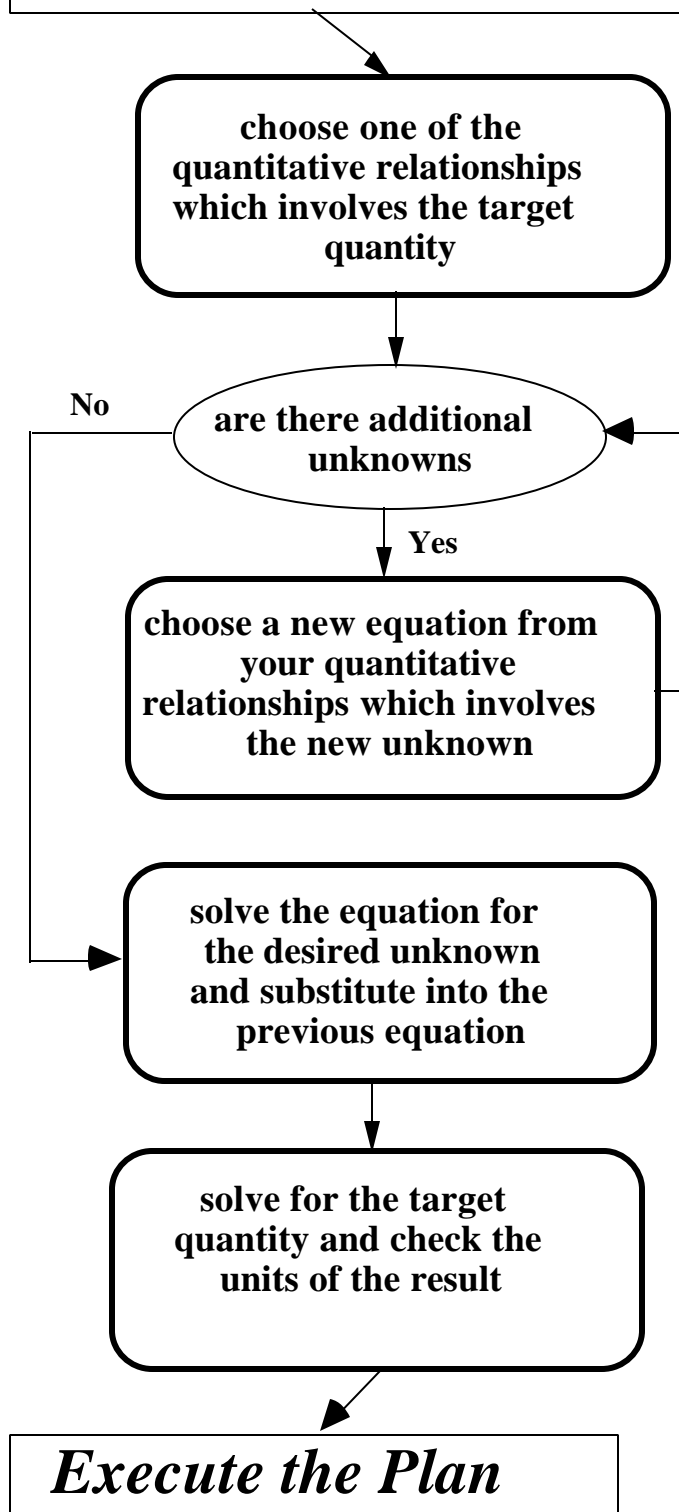
- What's going on?
 - What objects are involved?
 - What are they doing?
-
- Are all the important objects shown?
 - Are the spatial relations between the objects shown?
 - Are the important times represented?
 - Are the important motions represented?
 - Are the important interactions represented?
-
- Does the question ask about a specific measurable characteristic(s) about a particular object(s)? If not, reformulate it so it does.
-
- What is the system of interest?
 - Which physics principles could be used to solve the problem?
 - What information is really needed?
 - Are there only certain time intervals during which one approach is useful?
 - Should we make any approximations?

Describe the Physics



- What coordinate axes are useful? Which direction should we call positive?
- Relative to the coordinate axes, where is (are) the object(s) for each important time?
- Relative to the coordinate axes, what is (are) the velocity and acceleration for each object at each important time?
- Are other diagrams necessary to represent the interactions of each object or the time evolution of its state?
- What quantities are needed to define the problem mathematically using the approach chosen?
- Which symbols represent known quantities? Which symbols represent unknown quantities?
- Are all quantities having different values labeled with unique symbols?
- Does the diagram(s) have all of the essential information from the sketch?
- Which of the unknowns defined on the diagram(s) answers the question?
- What equations represent the general principles(s) specified in our approach and relate the physics quantities defined in the diagram?
- During what time intervals are those relationships either true or useful?
- Are there any equations that represent special conditions that are true for some quantities in this problem?

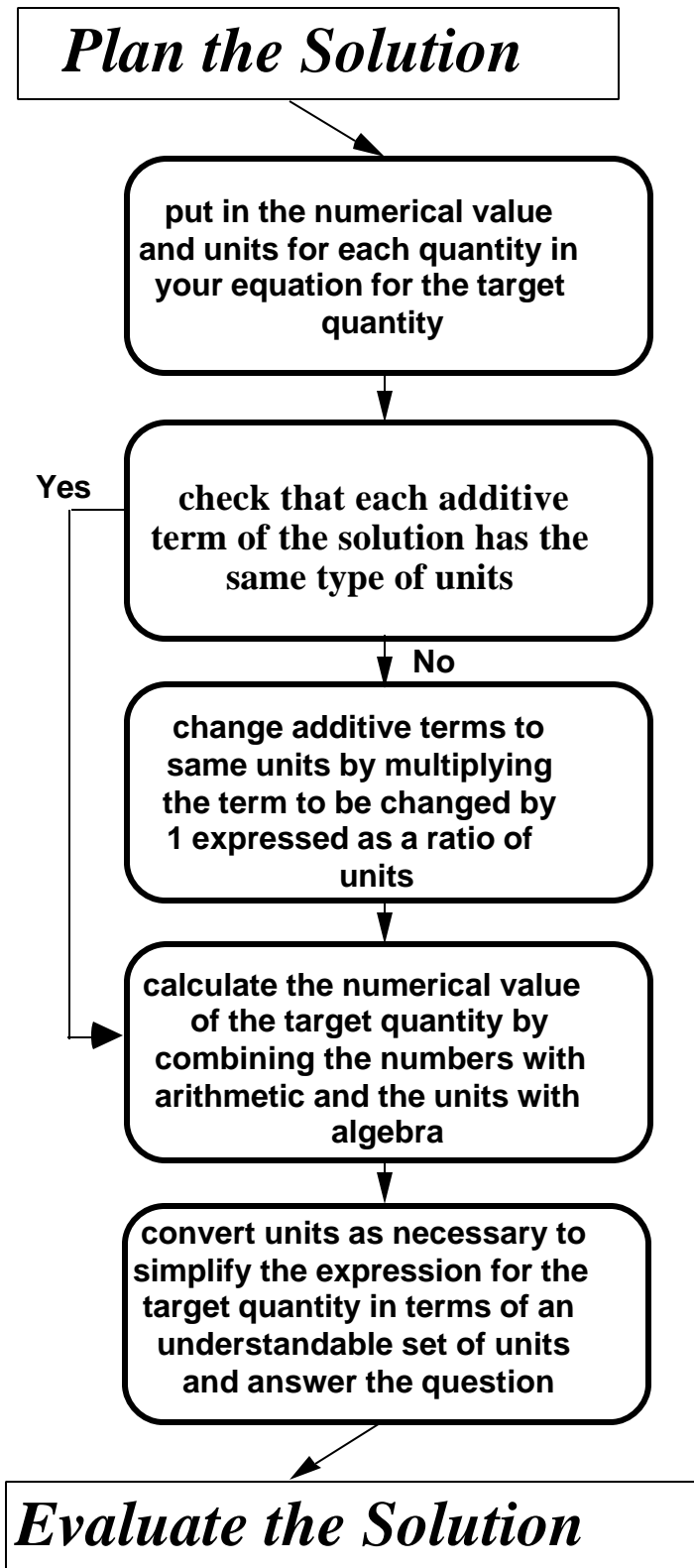
Describe the Physics



Plan the Solution

- Which quantitative relationship includes the target quantity?
 - For what object does that equation apply?
 - For what time interval does that equation apply?
- Are there any unknowns in the equation other than the target quantity?
 - Are there any unknowns that cancel out in the algebra?
- Which quantitative relationship includes the unknown quantity?
 - For what object does that equation apply?
 - For what time interval does that equation apply?
 - Is this equation different from those already used in this problem?
- What unknown is the target of this specific equation?
 - Which previous equations have that unknown?
 - Are there any quantities that cancel out in the algebra?
- After all the substitution for unknowns, is the only unknown left the target quantity?
 - Are the units the same on both sides of the equation?

Execute the Plan



- Which values (numbers with units) from the physics description should be put into the equation for the target quantity?

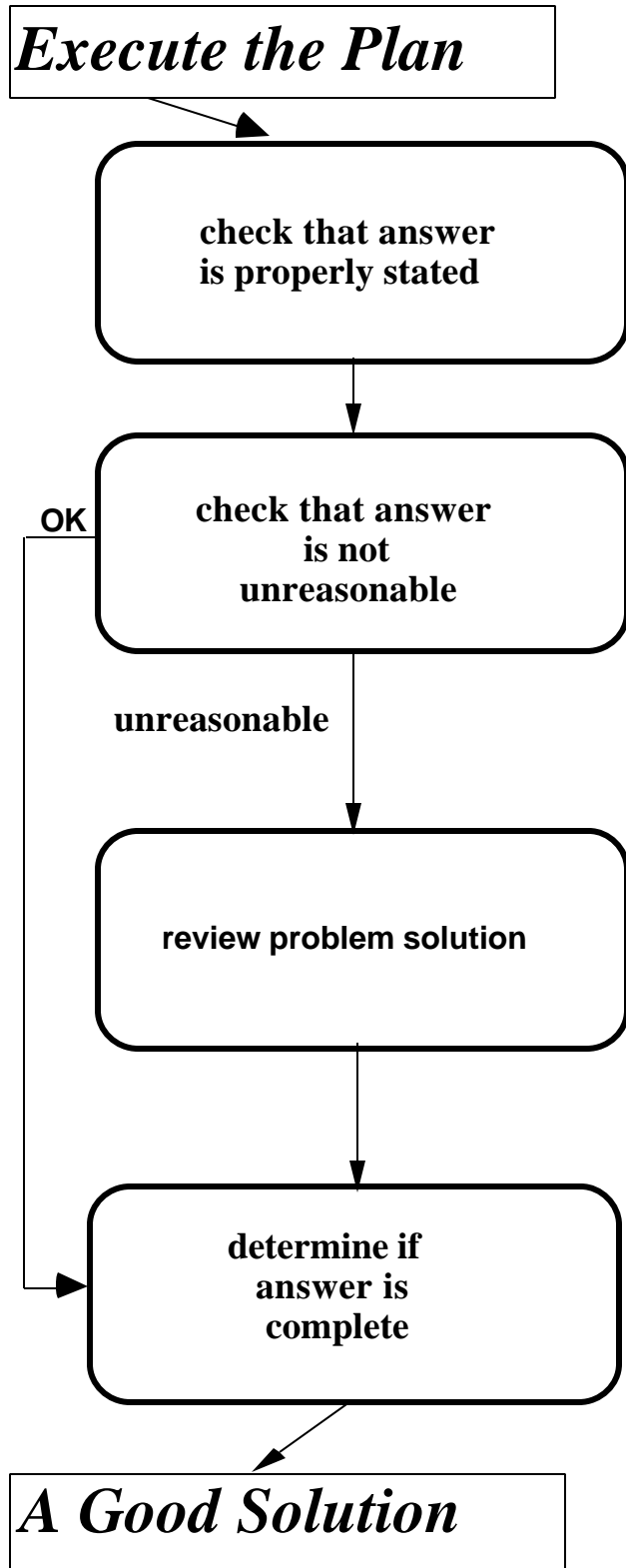
- Do you need to convert units?

- What ratio of units equals 1?

- Use a calculator for the numbers and algebra for the units.
- Do any units cancel?

- Do we need to convert any units?
- What is the most reasonable set of consistent units for this problem?

Evaluate the Answer



- Do the units make sense?
- Do vector quantities have both magnitude and direction?
- If someone else read just your answer, would they know what it meant?

- Does the answer fit with your mental picture of the situation?
- Is the answer the magnitude you would expect in this situation?
- Do you have any knowledge of a similar situation that you can compare with to see if the answer is reasonable?
- Can you change the situation (and thus your equation for the target quantity) to describe a simpler problem to which you know the answer?

- Is your physics description complete?
- Are the definitions of your physics quantities unique?
- Do the signs of your physics quantities agree with your coordinate system?
- Can you justify all of the mathematical steps in your solution plan?
- Did you use units in a consistent manner in your execution?
- Is there a calculation mistake in the execution?

- Have you answered the question from the Focus the Problem step?
- Could someone else read and follow the solution plan?
- Are you sure you can justify each mathematical step in the plan?

Format Sheet for Problem Solving

FOCUS the PROBLEM

Picture and Given Information

Question(s)

Approach

DESCRIBE the PHYSICS

Diagram(s) and Define Quantities

Target Quantity(ies)

Quantitative Relationships

PLAN the SOLUTION

Construct Specific Equations

EXECUTE the PLAN

Calculate Target Quantity(ies)

EVALUATE the ANSWER

Is Answer Properly Stated?

Is Answer Unreasonable?

Is Answer Complete?

(extra space if needed)

Check Units

Grading Feedback for Students' Problem Solutions

Focus on the Problem and Describe the Physics

Picture or Diagram is misleading or inaccurate

- missing important objects or interactions
- includes spurious objects or interactions
- other incorrect diagrammatic translations of problem information

Relevant variables not assigned and clearly labeled

- many important variables not defined
- defined variables not clearly distinguished from each other

Approach invalid, too vague, or missing

- application of principles inappropriate
- misunderstanding of fundamental principle
- simplifying approximations not stated or inappropriate

Necessary fundamental principles missing

Incorrect or invalid statement of known values or assumptions

Incorrect assertion of general relationships between variables

- application of principles to inappropriate parts of the problem
- incorrectly assumed relationship between unknown variables, such as $T_1=T_2$.
- overlooked important relationship between unknown variables, such as $a_1=a_2$.
- misunderstanding of fundamental principle

Incorrect statement of target variable or no target stated

- target variable doesn't correspond to question in Approach
- does not explicitly state target variable
- wrong target

Major misconception

Plan the Solution

Poor use of the physics description to generate a plan

- physics description was not used to generate a plan
- inappropriate equation(s) was introduced
- undefined variables used in equations

Improper construction of specific equations

- inappropriate substitution of variables into general equations
- numerical values were substituted too soon

Solution order is missing or unclear

- there is no clear logical progression through the problem
- solution order can't be understood from what is written

Plan can not be executed

- there are not enough equations
- a relationship was counted more than once

Execute the Plan and Evaluate the Solution

Execution is illogical

- Incorrect physics was introduced to solve the problem
- unacceptable mathematical assumption was used

Mistake in execution

- algebra mistake
- used incorrect values for known variables

Did not check units and/or sign of answer

Notes: