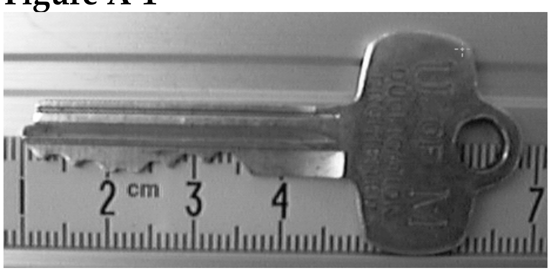
Appendix A: Significant Figures

Calculators make it possible to get an answer with a huge number of figures. Unfortunately many of them are meaningless! For instance if you needed to split $1.00 among three people, you could never give them each exactly $0.333333… The same is true for measurements. If you use a meter stick with millimeter markings to measure the length of a key, as in figure B-1, you could not measure more precisely than a quarter or half or a third of a mm. Reporting a number like 5.8132712 cm would not only be meaningless, it would be misleading.

Figure A-1



In your measurement, you can precisely determine the distance down to the nearest millimeter and then improve your precision by estimating the next figure. It is always assumed that the last figure in the number recorded is uncertain. So, you would report the length of the key as 5.81 cm. Since you estimated the 1, it is the uncertain figure. If you don't like estimating, you might be tempted to just give the number that you know best, namely 5.8 cm, but it is clear that 5.81 cm is a better report of the measurement. An estimate is always necessary to report the most precise measurement. When you quote a measurement, the reader will always assume that the last figure is an estimate. Quantifying that estimate is known as **estimating uncertainties**. Appendix B will illustrate how you might use those estimates to determine the uncertainties in your measurements.

What are significant figures?

The number of significant figures tells the reader the precision of a measurement. Table A-1 gives some examples. One of the things that this table illustrates is that not all zeros are significant. For example, the zero in front of 0.45 is not significant, while the 0 in 1.50 is.

Table A-1

Length (centimeters)	Number of Significant Figures
12.74	4
11.5	3
1.50	3
1.5	2
12.25345	7
0.8	1
0.05	1

A good rule is to always express your values in scientific notation. If you say that your friend lives 143 m from you, you are saying that you are sure of that distance to within a few meters (3 significant figures). What if you really only know the distance to a few tens of meters (2 significant figures)? Then you need to express the distance in scientific notation 1.4×10^2 m.

Is it always better to have more figures?

Consider the measurement of the length of the key Figure A-1 if we used a scale with ten etchings to every millimeter. Then we could use a microscope to measure the spacing to the nearest tenth of a millimeter and guess at the one hundredth millimeter. Our measurement could be 5.174 cm with the uncertainty in the last figure, four significant figures instead of three. This is because our improved scale allowed our estimate to be more precise. This added precision is shown by more significant figures. The more significant figures a number has, the more precise it is.

How do I use significant figures in calculations?

When using significant figures in calculations, you need to keep track of how the uncertainty propagates. There are mathematical procedures for doing this estimate in the most precise manner. This type of estimate depends on knowing the statistical distribution of your measurements. With a lot less effort, you can do a cruder estimate of the uncertainties in a calculated result. This crude method gives an overestimate of the uncertainty but it is a good place to start. For this course this simplified uncertainty estimate (described in Appendix B and below) will be good enough.

Addition and subtraction

Doing addition and subtraction is straightforward. Your result should have the number of significant figures as the least precise measurement. Some examples are given below. The uncertain figure in each number is shown in **bold-faced** type.

addition	subtraction
6.24**2**	5.87**5**
+4.2**3**	-3.3**4**
+0.01**3**	2.5**35**
10.4**85**	
10.4**9**	2.5**4**

The examples above illustrate some rules of working with numbers. First, do all calculations with as many significant figures as you have. Then, at the end of the calculations, round the number off to the correct number of significant figures for the answer. Notice that we had to round the uncertain figure.

Multiplication and division

Multiplication and division are more complicated than addition. The common rule, keeping the same number of significant figures in your answer as in the starting number with the least significant figures, may not always work. As shown in the examples. However, this is the quickest and best rule to use. When in doubt, you can keep track of the significant figures in the calculation as is done in the examples.

multiplication	
15.8**4**	17.2**7**
x 2.**5**	x 4.**0**
7920	69.**080**
316**8**	
3**9.600**	
4**0**	69.

division	
1**17**	2**5**
2**3**)269**1**	7**5**)187**5**
2**3**	15**0**
39	3**75**
23	**375**
161	
161	
1.**2** x 10^2	2.**5** x 10^1

PRACTICE EXERCISES

1. Determine the number of significant figures in the following table.

Length (centimeters)	Number of Significant Figures
17.87	
0.4730	
17.9	
0.473	
18	
0.47	
1.34×10^2	
2.567×10^5	
2.0×10^{10}	
1.001	
1.000	
1	
1000	
1001	

2. Add: 121.3 to 6.7×10^2:

Answer: $121.3 + 6.7 \times 10^2 = 7.9 \times 10^2$

3. Multiply: 34.2 and 1.5×10^4

Answers: $34.2 \times 1.5 \times 10^4 = 5.1 \times 10^5$