Appendix B: Accuracy, Precision and Uncertainty

How tall are you? How old are you? When you answered these everyday questions, you probably did it in round numbers such as "five foot, six inches" or "nineteen years, three months." But how true are these answers? Are you exactly 5' 6" tall? Probably not. You estimated your height at 5’ 6" and just reported two significant figures. Typically, you round your height to the nearest inch, so that your actual height falls somewhere between 5' 5½" and 5' 6½" tall, or 5' 6" ± ½". This ± ½" is the **uncertainty,** and it informs the reader of the precision of the **value** 5' 6".

What is uncertainty?

Whenever you measure something, there is always some uncertainty. There are two categories of uncertainty: **systematic** and **random**.

(1) **Systematic uncertainties** are those that consistently cause the value to be too large or too small. Systematic uncertainties include such things as reaction time, inaccurate meter sticks, optical parallax and miscalibrated balances. In principle, systematic uncertainties can be eliminated if you know they exist.

(2) **Random uncertainties** are variations in the measurements that occur without a predictable pattern. If you make precise measurements, these uncertainties arise from the estimated part of the measurement. Random uncertainty can be reduced, but never eliminated. We need a technique to report the contribution of this uncertainty to the measured value.

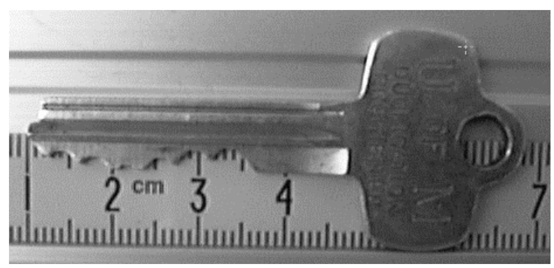
How do I determine the uncertainty?

This Appendix will discuss two basic techniques for determining the uncertainty: **estimating the uncertainty** and measuring the **average deviation.** Which one you choose will depend on your need for precision. If you need a precise determination of some value, the best technique is to measure that value several times and use the average deviation as the uncertainty. Examples of finding the average deviation are given below.

How do I estimate uncertainties?

If time or experimental constraints make repeated measurements impossible, then you will need to estimate the uncertainty. When you estimate uncertainties you are trying to account for anything that might cause the measured value to be different if you were to take the measurement again. For example, suppose you were trying to measure the length of a key, as in Figure B-1.

Figure B-1



If the true value were not as important as the magnitude of the value, you could say that the key’s length was 6cm, give or take 1cm. This is a crude estimate, but it may be acceptable. A better estimate of the key’s length, as you saw in Appendix A, would be 5.81cm. This tells us that the worst our measurement could be off is a fraction of a mm. To be more precise, we can estimate it to be about a third of a mm, so we can say that the length of the key is 5.81 ± 0.03 cm.

Another time you may need to estimate uncertainty is when you analyze video data. Figures B-2 and B-3 show a ball rolling off the edge of a table. These are two consecutive frames, separated in time by 1/30 of a second.

Figure B-2

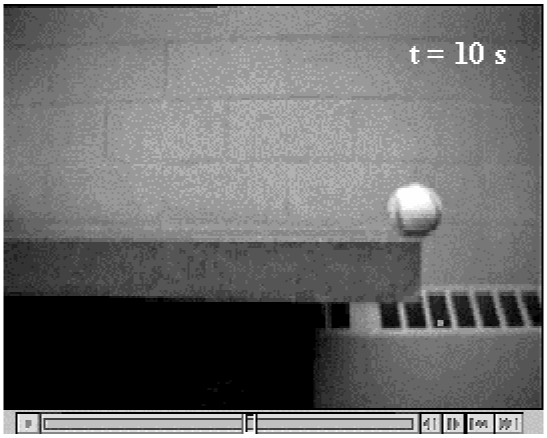
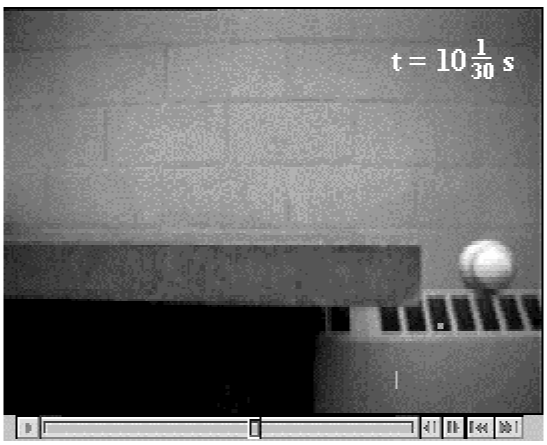


Figure B-3



The exact moment the ball left the table lies somewhere between these frames. We can estimate that this moment occurs midway between them ($t = 10\frac{1}{60}s$). Since it must occur at some point between them, the worst our estimate could be off by is $\frac{1}{60}s$. We can therefore say the time the ball leaves the table is $t = 10\frac{1}{60} \pm \frac{1}{60}s$.

How do I find the average deviation?

If estimating the uncertainty is not good enough for your situation, you can experimentally determine the un-certainty by making several measure-ments and calculating the average deviation of those measurements. To find the average deviation: (1) Find the average of all your measurements; (2) Find the absolute value of the difference of each measurement from the average (its deviation); (3) Find the average of all the deviations by adding them up and dividing by the number of measurements. Of course you need to take enough measure-ments to get a distribution for which the average has some meaning.

In example 1, a class of six students was asked to find the mass of the same penny using the same balance. In example 2, another class measured a different penny using six different balances. Their results are listed below:

Class 1: Penny A massed by six different students on the same balance.

Mass (grams)
3.110
3.125
3.120
3.126
3.122
3.120
3.121 average.
The deviations are: 0.011g, 0.004g, 0.001g, 0.005g, 0.001g, 0.001g
Sum of deviations: 0.023g
Average deviation: (0.023g) / 6 = 0.004g
Mass of penny A: 3.121 ± 0.004g

Class 2: Penny B massed by six different students on six different balances

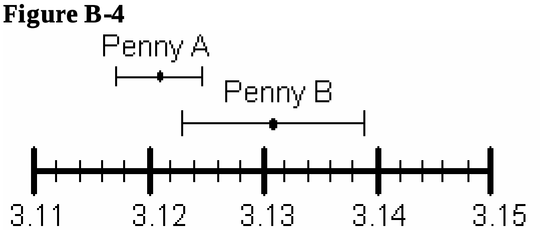
Mass (grams)
3.140
3.133
3.144
3.118
3.126
3.125
3.131 average
The deviations are: 0.009g, 0.002g, 0.013g, 0.013g, 0.005g, 0.006g
Sum of deviations: 0.048g
Average deviation: (0.048g) / 6= 0.008g
Mass of penny B: 3.131 ± 0.008g

However you choose to determine the uncertainty, you should always state your method clearly in your report. For the remainder of this appendix, we will use the results of these two examples.

How do I know if two values are the same?

If we compare only the average masses of the two pennies we see that they are different. But now include the uncertainty in the masses. For penny A, the most likely mass is somewhere between 3.117g and 3.125g. For penny B, the most likely mass is somewhere between 3.123g and 3.139g. If you compare the ranges of the masses for the two pennies, as shown in Figure B-4, they just overlap. Given the uncertainty in the masses, we are able to conclude that the masses of the two pennies could be the same. If the range of the masses did not overlap, then we ought to conclude that the masses are probably different.

Figure B-4



Mass of pennies (in grams) with uncertainties

Which result is more precise?

Suppose you use a meter stick to measure the length of a table and the width of a hair, each with an uncertainty of 1 mm. Clearly you know more about the length of the table than the width of the hair. Your measurement of the table is very precise but your measurement of the width of the hair is rather crude. To express this sense of precision, you need to calculate the percentage uncertainty. To do this, divide the uncertainty in the measurement by the value of the measurement itself, and then multiply by 100%. For example, we can calculate the precision in the measurements made by class 1 and class 2 as follows:

Precision of Class 1's value:
(0.004 g ÷ 3.121 g) x 100% = 0.1 %
Precision of Class 2's value:
(0.008 g ÷ 3.131 g) x 100% = 0.3 %

Class 1's results are more precise. This should not be surprising since class 2 introduced more uncertainty in their results by using six different balances instead of only one.

Which result is more accurate?

Accuracy is a measure of how your measured value compares with the real value. Imagine that class 2 made the measurement again using only one balance. Unfortunately, they chose a balance that was poorly calibrated. They analyzed their results and found the mass of penny B to be 3.556 ± 0.004 g. This number is more precise than their previous result since the uncertainty is smaller, but the new measured value of mass is very different from their previous value. We might conclude that this new value for the mass of penny B is different, since the range of the new value does not overlap the range of the previous value. However, that conclusion would be **wrong** since our uncertainty has not taken into account the inaccuracy of the balance. To determine the accuracy of the measurement, we should check by measuring something that is known. This procedure is called calibration, and it is absolutely necessary for making accurate measurements.

Be cautious! It is possible to make measurements that are extremely precise and, at the same time, grossly inaccurate.

How can I do calculations with values that have uncertainty?

When you do calculations with values that have uncertainties, you will need to estimate (by calculation) the uncertainty in the result. There are mathematical techniques for doing this, which depend on the statistical properties of your measurements. A very simple way to estimate uncertainties is to find the *largest possible uncertainty* the calculation could yield. **This will always overestimate the uncertainty of your calculation**, but an overestimate is better than no estimate. The method for performing arithmetic operations on quantities with uncertainties is illustrated in the following examples:

Addition:

(3.131 ± 0.008 g) + (3.121 ± 0.004 g) = ?

First, find the sum of the values:

3.131 g + 3.121 g = 6.252 g

Next, find the largest possible value:

3.139 g + 3.125 g = 6.264 g

The uncertainty is the difference between the two:

6.264 g – 6.252 g = 0.012 g

Answer: 6.252 ± 0.012 g.

Note: This uncertainty can be found by simply adding the individual uncertainties:

0.004 g + 0.008 g = 0.012 g

Multiplication:

(3.131 ± 0.013 g) x (6.1 ± 0.2 cm) = ?

First, find the product of the values:

3.131 g x 6.1 cm = 19.1 g-cm

Next, find the largest possible value:

3.144 g x 6.3 cm = 19.8 g-cm

The uncertainty is the difference between the two:

19.8 g-cm - 19.1 g-cm = 0.7 g-cm

Answer: 19.1 ± 0.7g-cm.

Note: The percentage uncertainty in the answer is the sum of the individual percentage uncertainties:

$$\frac{0.013}{3.131}\times 100\% + \frac{0.2}{6.1}\times 100\% = \frac{0.7}{19.1}\times 100\%$$

Subtraction:

(3.131 ± 0.008 g) – (3.121 ± 0.004 g) = ?

First, find the difference of the values:

3.131 g - 3.121 g = 0.010 g

Next, find the largest possible difference:

3.139 g – 3.117 g = 0.022 g

The uncertainty is the difference between the two:

0.022 g – 0.010 g = 0.012 g

Answer: 0.010±0.012 g.

Note: This uncertainty can be found by simply adding the individual uncertainties:

0.004 g + 0.008 g = 0.012 g

Notice also, that zero is included in this range, so it is possible that there is no difference in the masses of the pennies, as we saw before.

Division:

(3.131 ± 0.008 g) ÷ (3.121 ± 0.004 g) = ?

First, divide the values:

3.131 g ÷ 3.121 g = 1.0032

Next, find the largest possible value:

3.139 g ÷ 3.117 g = 1.0071

The uncertainty is the difference between the two:

1.0071 - 1.0032 = 0.0039

Answer: 1.003 ± 0.004

Note: The percentage uncertainty in the answer is the sum of the individual percentage uncertainties:

$$\frac{0.008}{3.131}\times 100\% + \frac{0.004}{3.121}\times 100\% = \frac{0.0039}{1.0032}\times 100\%$$

Notice also, the largest possible value for the numerator and the smallest possible value for the denominator gives the largest result.

The same ideas can be carried out with more complicated calculations. Remember this will always give you an overestimate of your uncertainty. There are other calculation techniques, which give better estimates for uncertainties. If you wish to use them, please discuss it with your instructor to see if they are appropriate.

These techniques help you estimate the random uncertainty that always occurs in measurements. They will not help account for mistakes or poor measurement procedures. There is no substitute for taking data with the utmost of care. A little forethought about the possible sources of uncertainty can go a long way in ensuring precise and accurate data.

PRACTICE EXERCISES:

B-1. Consider the following results for different experiments. Determine if they agree with the accepted result listed to the right. Also calculate the precision for each result.

a) $g = 10.4 \pm 1.1\ m/s^2$ — $g = 9.8\ m/s^2$

b) $T = 1.5 \pm 0.1$ sec — $T = 1.1$ sec

c) $k = 1368 \pm 45\ N/m$ — $k = 1300 \pm 50\ N/m$

Answers: a) Yes, 11%; b) No, 7%; c) Yes, 3.3%

B-2. The area of a rectangular metal plate was found by measuring its length and its width. The length was found to be 5.37 ± 0.05 cm. The width was found to be 3.42 ± 0.02 cm. What is the area and the average deviation?

Answer: $18.4 \pm 0.3\ cm^2$

B-3. Each member of your lab group weighs the cart and two mass sets twice. The following table shows this data. Calculate the total mass of the cart with each set of masses and for the two sets of masses combined.

Cart (grams)	**Mass set 1** (grams)	**Mass set 2** (grams)
201.3	98.7	95.6
201.5	98.8	95.3
202.3	96.9	96.4
202.1	97.1	96.2
199.8	98.4	95.8
200.0	98.6	95.6

Answers:

Cart and set 1:	299.3±1.6 g.
Cart and set 2:	297.0±1.2 g.
Cart and both sets:	395.1±1.9 g.