LABORATORY VI
ROTATIONAL DYNAMICS

So far this semester, you have been asked to think of objects as point particles rather than extended bodies. This assumption is useful and sometimes good enough. However, the approximation of extended bodies as point particles gives an incomplete picture of the real world.

Now we begin a more realistic description of the motion and interactions of objects. Real objects usually rotate as well as move along trajectories. You already have a lot of experience with rotating objects from your everyday life. Every time you open or close a door, something rotates. Rotating wheels are everywhere. Balls spin when they are thrown. The earth rotates about its axis. Rotations are important whether you are discussing galaxies or subatomic particles.

An object's rotational motion can be described with the kinematic quantities you have already used: position, velocity, acceleration, and time. In these problems, you will explore the connection of these familiar linear kinematic quantities to a more convenient set of quantities for describing rotational kinematics: angle, angular velocity, angular acceleration, and time. Often, the analysis of an interaction requires the use of both linear and rotational kinematics.

OBJECTIVES:

Successfully completing this laboratory should enable you to:

- Use linear kinematics to predict the outcome of a rotational system.
- Choose a useful system when using rotational kinematics.
- Identify links between linear and rotational motion.
- Decide when rotational kinematics is more useful and when it is a not.
- Use both linear and rotational kinematics as a means of describing the behavior of systems.

PREPARATION:

Read Paul M. Fishbane: Chapter 9, Section 9-1; Chapter 5, Section 5-4. You should also be able to:

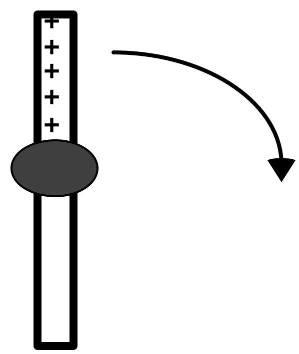
- Analyze the motion of an object using linear kinematics and video analysis.
- Identify the connection between the linear description of motion and rotational description of motion
- Calculate the angle of rotation in radians.
- Calculate the angular speed of a rotating object.
- Calculate the angular acceleration of a rotating object.

PROBLEM #1: ANGULAR SPEED AND LINEAR SPEED

You are working with an engineering group testing equipment that might be used on a satellite. To equalize the heat load from the sun, the satellite will spin about its center. Your task is to determine the forces exerted on delicate measuring equipment when the satellite spins at a constant angular speed. You know that since any object traveling in a circular path must have exerted on it a non-zero net force, that object must be accelerating. As a first step in finding the net force, you decide to calculate the linear speed of any object in the satellite as a function of its distance from the center of the satellite and the satellite's angular speed. From the linear speed of the object in circular motion, you calculate its acceleration. You will test your calculations in a laboratory on earth, before launching the satellite.

EQUIPMENT

You will use an apparatus that spins a horizontal beam on an A-frame base. A top view of the device is shown to the right. You will also have a meter stick, a stopwatch, and the video analysis equipment.



PREDICTION

What are you trying to calculate? Restate the problem to clearly identify your objective. Illustrate

WARM UP

Read: Fishbane Chapter 5, section 5.4 and Chapter 9, section 9.1.

The following questions will help you to reach your prediction and the analysis of your data.

1. Draw the trajectory of a point on a beam rotating. Choose a coordinate system. Choose a point on that trajectory that is not on a coordinate axis. Draw vectors representing the position, velocity, and acceleration of that point.

2. Write equations for each component of the position vector, as a function of the distance of the point from the axis of rotation and the angle the vector makes with an axis of your coordinate system. Next, calculate how that angle depends on time and the constant angular speed of the beam. Sketch three graphs, (one for each of these equations) as a function of time. Explain why one of the graphs increases monotonically with time, but the other two oscillate.

3. Using your equations for components of the position of the point, calculate an equation for each component of the velocity of the point. Graph these two equations as a function of time. Compare these graphs to those for the components of the position of the object (when one component of the position is at a maximum, for example, is the same component of the velocity at a maximum value?) Draw these components at the point you have chosen in your drawing; verify that their vector sum gives the correct direction for the velocity of the point.
4. Use your equations for the point's velocity components to calculate its speed. Does the speed change with time? Should it?
5. Use the equations for the point's velocity components to calculate an equation for each component of the point's acceleration. Graph these two equations as functions of time, and compare to the velocity and position graphs. Verify that the vector sum of the components gives the correct direction for the acceleration of the point you have chosen in your drawing. Use the acceleration components to calculate the magnitude of the acceleration.
6. For comparison, write down the expression for the acceleration of the point as a function of its speed and its distance from the axis of rotation.

EXPLORATION

Practice spinning the beam at different angular speeds. How many rotations does the beam make before it slows down appreciably? Select a range of angular speeds to use in your measurements.

Move the apparatus to the floor and adjust the camera tripod so that the camera is directly above the middle of the spinning beam. Make sure the beam is level. Practice taking some videos. Find the best distance and angle for your video. How will you make sure that you always measure the same position on the beam?

Plan how you will measure the perpendicular components of the velocity to calculate the speed of the point. How will you also use your video to measure the angular speed of the beam?

MEASUREMENT

Take a video of the spinning beam. Be sure you have more than two complete revolution of the beam. For best results, use the beam itself when calibrating your video.

Determine the time it takes for the beam to make two complete revolutions and the distance between the point of interest and the axis of rotation. Set the scale of your axes appropriately so you can see the data as it is digitized.

Decide how many different points you will measure to test your prediction. How will you ensure that the angular speed is the same for all of these measurements?

How many times will you repeat these measurements using different angular speeds?

ANALYSIS

Analyze your video by following a single point on the beam for at least two complete revolutions. Use the velocity components to determine the direction of the velocity vector. Is it in the expected direction?

Analyze enough different points in the same video to make a graph of speed of a point as a function of distance from the axis of rotation. What quantity does the slope of this graph represent?

Calculate the acceleration of each point and graph the acceleration as a function of the distance from the axis of rotation. What quantity does the slope of this graph represent?

CONCLUSIONS

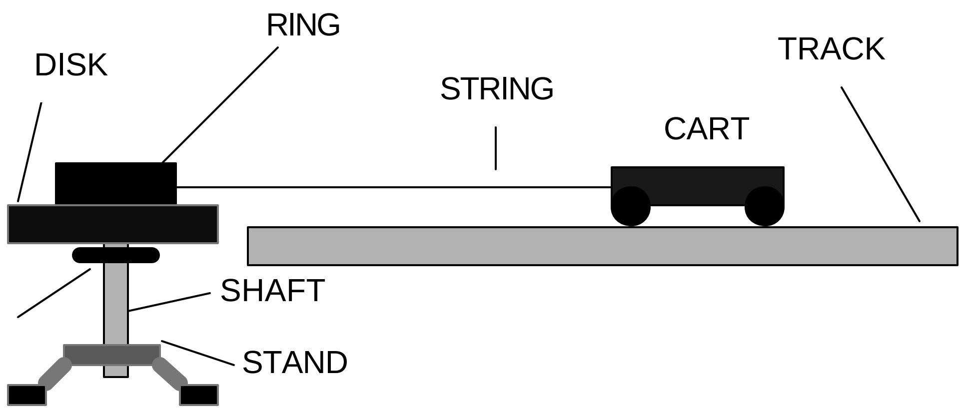
How do your results compare to your predictions and the answers to the warm up questions? Did the measured acceleration match the acceleration predicted by your equation from Warm up question 5? Question 6? Explain.

Was the measured linear speed of each point on the beam a constant? Demonstrate this in terms of your fit equations for velocity.

PROBLEM#2:
ROTATION AND LINEAR MOTION AT CONSTANT SPEED

While helping a friend take apart a lawn mower engine, you notice the pull cord wraps around a heavy, solid disk, "a flywheel," and that disk is attached to a shaft. You know that the flywheel must have at least a minimum angular speed to start the engine. Intrigued by this setup, you wonder how the angular speed of the flywheel is related to the speed of the handle at the end of the pull cord, and you make a prediction. To test your prediction, you make a laboratory model so that you can measure the speed of the cord, the speed of the point on the flywheel where the cord is attached, and the angular speed of the flywheel.

EQUIPMENT



You will have a disk mounted horizontally on a sturdy stand, with a ring coaxially fastened on the disk. Together, the disk and the ring represent the flywheel. The disk and the ring rotate freely about a vertical shaft through their center.

You will attach one end of a string to the outside surface of the ring, so that it can wrap around the ring. The other end of the string will be connected to a cart that can move along a level track. You also have a stopwatch, a meter stick, an end stop, some wooden blocks and the video analysis equipment.

PREDICTION

Restate the problem. What are you trying to calculate? Which experimental parameters will be determined by the laboratory equipment, and which ones will you control?

WARM UP

Read: Fishbane Chapter 5, section 5.4 and Chapter 9, section 9.1.

The following questions will help you to reach your prediction.

1. Draw a top view of the system. Draw the velocity and acceleration vectors of a point on the outside edge of the ring. Draw a vector representing the angular velocity of the ring. Draw the velocity and acceleration vectors of a point along the string. Draw the velocity and acceleration vectors of the cart. Write an equation for the relationship between the linear velocity of the point where the string is attached to the ring and the velocity of the cart (if the string is taut).

2. Choose a coordinate system useful for describing the motion of the point where the string is attached to the ring. Select a point on the outside edge of the ring. Write equations for the perpendicular components of the position vector as a function of the distance from the axis of rotation and the angle the vector makes with one axis of your coordinate system. Calculate how that angle depends on time and the constant angular speed of the ring. Sketch three graphs, (one for each of these equations) as a function of time.

3. Using your equations for the components of the position of the point, determine equations for the components of the velocity of the point. Graph these equations as a function of time. Compare these graphs to those representing the components of the position of the object.

4. Use your equations for the components of the velocity of the point to calculate its speed. Is the speed a function of time or is it constant?

5. Now write an equation for the cart's speed as a function of time, assuming the string is taut.

EXPLORATION

Try to make the cart move along the track with a constant velocity. (To account for friction, you may need to slant the track slightly. You might even use some quick video analysis to get this right.) Do this before you attach the string.

Try two different ways of having the string and the cart move with the same constant velocity so that the string remains taut. Try various speeds and pick the way that works most consistently for you. If the string goes slack during the measurement you must redo it.

(1) Gently push the cart and let it go so that the string unwinds from the ring at a constant speed.
(2) Gently spin the disk and let it go so that the string winds up on the ring at a constant speed.

Where will you place the camera to give the best recording looking down on the system? You will need to get data points for both the motion of the ring and the cart. Try some test runs.

Decide what measurements you need to make to determine the speed of the outer edge of the ring and the speed of the string from the same video.

Outline your measurement plan.

MEASUREMENT

Make a video of the motion of the cart **and** the ring for several revolutions of the ring. Measure the radius of the ring. What are the uncertainties in your measurements? (See Appendices A and B if you need to review how to determine significant figures and uncertainties.)

Analyze your video to determine the velocity of the cart and, because the string was taut throughout the measurement, the velocity of the string. Use your measurement of the distance the cart goes and the time of the motion to choose the scale of the computer graphs so that the data is visible when you take it. If the velocity was not constant, adjust your equipment and repeat the measurement.

Analyze the same video to determine the velocity components of the edge of the ring. Use your measurement of the diameter of the ring and the time of the motion to choose the scale of the computer graphs so that the data is visible when you take it.

In addition to finding the angular speed of the ring from the speed of the edge and the radius of the ring, also determine the angular speed directly (using its definition) from either position component of the edge of the ring versus time graph.

ANALYSIS

Use an analysis technique that makes the most efficient use of your data and your time.

Compare the measured speed of the edge of the ring with the measured speed of the cart and thus the string. Calculate the angular speed of the ring from the measured speed of the edge of the ring and the distance of the edge of the ring from the axis of rotation. Compare that to the angular speed measured directly.

CONCLUSIONS

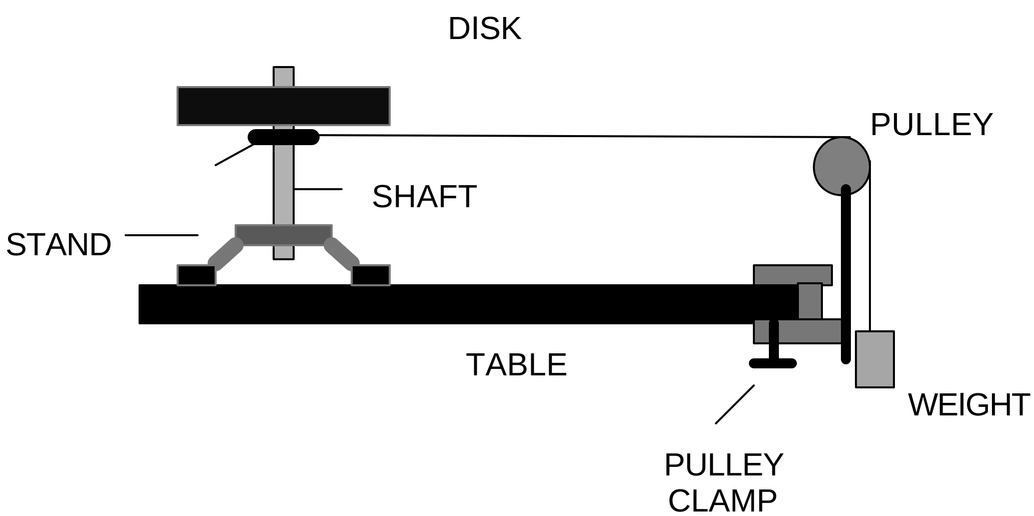
Did your measurements agree with your initial prediction? Why or why not? What are the limitations on the accuracy of your measurements and analysis?

Explain why it is difficult to keep the string taut in this measurement, by considering the forces exerted on each end of the string? Determine the force of the string on the cart and the force of the cart on the string. Determine the force of the string on the ring and the force of the ring on the string. What is the string tension?

PROBLEM #3: ANGULAR AND LINEAR ACCELERATION

You are working in a bioengineering laboratory when the building power fails. An ongoing experiment will be damaged if there is any temperature change. There is a gasoline powered generator on the roof for just such emergencies. You run upstairs and start the generator by pulling on a cord attached to a flywheel. It is such hard work that you begin to design a gravitational powered generator starter. The generator you design has its flywheel as a horizontal disk that is free to rotate about its center. One end of a rope is wound up on a horizontal ring attached to the center of the flywheel. The free end of the rope goes horizontally to the edge of the building roof, passes over a vertical pulley, and then hangs straight down. A heavy block is attached to the hanging end of the rope. When the power fails, the block is released, the rope unrolls from the ring giving the flywheel a large enough angular acceleration to start the generator. To see if this design is feasible you must determine the relationship between the angular acceleration of the flywheel, the downward acceleration of the block, and the radius of the ring. Before putting more effort in the design, you test your idea by building a laboratory model of the device.

EQUIPMENT



You will have a disk mounted horizontally on a sturdy stand. The disk represents the flywheel and is free to rotate about a vertical shaft through its center. You will attach the string to a spool under the center of the disk, in place of the ring in your flywheel design. A string has one end wrapped around the horizontal spool. The other end of the string passes over a vertical pulley lined up with the tangent to the spool. An object (a mass hanger) is hung from the free end of the string so that it can fall past the table. You also have a stopwatch, a meter stick, a pulley clamp, a mass set, a mass hanger and the video analysis equipment.

PREDICTION

Reformulate the problem in your own words to understand its target. What do you need to calculate?

WARM UP

Read: Fishbane Chapter 5, section 5.4 and Chapter 9, section 9.1.

The following questions are designed to help you solve the problem in an organized way.

1. Draw a top view of the system. Draw the velocity and acceleration vectors of a point on the outside edge of the spool. Draw a vector representing the angular acceleration of the spool. Draw the velocity and acceleration vectors of a point along the string.

2. Draw a side view of the system. Draw the velocity and acceleration vectors of the hanging object. What is the relationship between the linear acceleration of the string and the acceleration of the hanging object if the string is taut? Do you expect the acceleration of the hanging object to be constant? Explain.

3. Choose a coordinate system useful to describe the motion of the spool. Select a point on the outside edge of the spool. Write equations giving the perpendicular components of the point's position vector as a function of the distance from the axis of rotation and the angle the vector makes with one axis of your coordinate system. Assume the angular acceleration is constant and that the disk starts from rest. Determine how the angle between the position vector and the coordinate axis depends on time and the angular acceleration of the spool. Sketch three graphs, (one for each of these equations) as a function of time.

4. Using your equations for components of the position of the point, calculate the equations for the components of the velocity of the point. Is the *speed* of this point a function of time or is it constant? Graph these equations as a function of time.

5. Use your equations for the components of the velocity of the point on the edge of the spool to calculate the components of the *acceleration* of that point. From the components of the acceleration, calculate the *square of the total acceleration* of that point. It looks like a mess but it can be simplified to two terms if you can use: $\sin^2(z)+\cos^2(z) = 1$.

6. From step 5, the magnitude acceleration of the point on the edge of the spool has one term that depends on time and another term that does not. Identify the term that depends on time by using the relationship between the angular speed and the angular acceleration for a constant angular acceleration. If you still don't recognize this term, use the relationship among angular speed, linear speed and distance from the axis of rotation. Now identify the relationship between this time-dependent term and the centripetal acceleration.

7. We also can solve the acceleration vector of the point on the edge of the spool into two perpendicular components by another way. One component is the centripetal acceleration and the other component is the tangential acceleration. In step 6, we already identify the centripetal acceleration term from the total acceleration. So now you can recognize the tangential acceleration term. How is the tangential acceleration of the edge of the spool related to the angular acceleration of the spool and the radius of the spool? What is the relationship between the angular acceleration of the spool and the angular acceleration of the disk?

8. How is the tangential acceleration of the edge of the spool related to the acceleration of the string? How is the acceleration of the string related to the acceleration of the hanging object? Explain the relationship between the angular acceleration of the disk and the acceleration of the hanging object.

9. Use the simulation “Lab7Sim” (See *Appendix F* for a brief explanation of how to use the simulations) to explore connections among graphs of angle, angular velocity, and angular acceleration for the flywheel; position, velocity, and acceleration for the hanging object; and position components of a point on the ring and the velocity components,. Adjust the sizes and masses of the objects to get clear graphs for at least two rotations of the disk. You may need to adjust the axis limits for the graphs, and redraw them several times.

EXPLORATION

Practice gently spinning the spool / disk system by hand. How long does it take the disk to stop rotating about its central axis? What is the average angular acceleration caused by this friction? Make sure the angular acceleration you use in your measurements is much larger than the one caused by friction so that it has a negligible effect on your results.

Find the best way to attach the string to the spool. How much string should you wrap around the spool? How should the pulley be adjusted to allow the string to unwind smoothly from the spool and pass over the pulley? Practice releasing the hanging object and the spool / disk system.

Determine the best mass to use for the hanging object. Try a large range. What mass will give you the smoothest motion? What is the highest angular acceleration? How many useful frames for a single video?

Where will you place the camera to give the best top view recording on the whole system? Try some test runs. Since you can’t get a video of the falling object and the top of the spinning spool / disk at the same time, attach a piece of tape to the string. The tape will show up in the video and will have the same linear motion as the falling object.

Decide what measurements you need to make to determine the angular acceleration of the disk and the acceleration of the string from the same video.

Outline your measurement plan.

MEASUREMENT

Make a video of the motion of the tape on the string **and** the disk for several revolutions. Measure the radius of the spool. What are the uncertainties in your measurements?

Digitize your video to determine the acceleration of the string and, because the string was taut throughout the measurement, the acceleration of the hanging object. Use your measurement of the distance and time that the hanging object falls to choose the scale of the computer graphs so that the data is visible when you take it. Check to see if the acceleration is constant.

Use a stopwatch and meter stick to directly determine the acceleration of the hanging object.

Digitize the same video to determine the velocity components of the edge of the disk. Use your measurement of the diameter of the disk and the time of the motion to choose the scale of the computer graphs so that the data is visible when you take it.

ANALYSIS

From an analysis of the video data for the tape on the string, determine the acceleration of the piece of tape on the string. Compare this acceleration to the hanging object's acceleration determined directly. Be sure to use an analysis technique that makes the most efficient use of your data and your time.

From your video data for the disk, determine if the angular speed of the disk is constant or changes with time.

Use the equations that describe the measured components of the velocity of a point at the edge of the disk to calculate the tangential acceleration of that point and use this tangential acceleration of the edge of the disk to calculate the angular acceleration of the disk (it is also the angular acceleration of spool). You can refer to the Warm up questions.

CONCLUSION

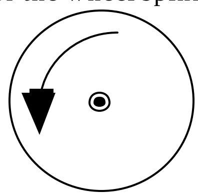
Did your measurements agree with your initial prediction? Why or why not? What are the limitations on the accuracy of your measurements and analysis?

Explain why it is not difficult to keep the string taut in this measurement by considering the forces exerted on each end of the string? Determine the pull of the string on the hanging object and the pull of the hanging object on the string, in terms of the acceleration of the hanging object. Determine the force of the string on the spool and the force of the spool on the string. What is the string tension? Is it equal to, greater than, or less than the weight of the hanging object?

☑ CHECK YOUR UNDERSTANDING

1. The direction of angular velocity for the wheel spinning as shown is:

   (a) Clockwise.
   (b) Counterclockwise.
   (c) Into the page.
   (d) Out of the page.
   (e) Down.

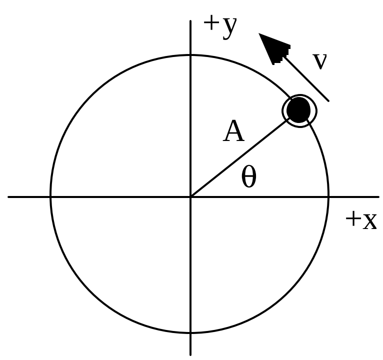


2. The wheel in question 1 is slowing down. The direction of its angular acceleration is:

   (a) Clockwise.
   (b) Counterclockwise.
   (c) Into the page.
   (d) Out of the page.
   (e) Down.

3. An object in circular motion has a constant angular velocity ? and is moving as shown. The object is at position x =+A, y=0 at time t=0. The y component of the object's **acceleration** is given by:

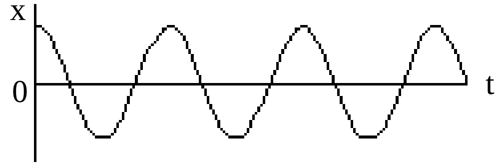
   (a) A ?^{2} cos (? t).
   (b) -A ?^{2} sin (? t).
   (c) A ?^{2}.
   (d) A ? cos (? t).
   (e) -A ? sin (? t).



4. If an object is in circular motion, its

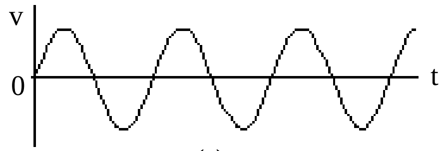
   (a) Velocity is proportional to its displacement.
   (b) Acceleration is proportional to its displacement.
   (c) Both a and b.
   (d) Neither a nor b.

The graph represents the x-component of the position of an object in circular motion.

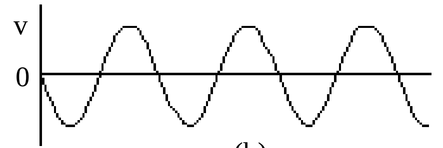


☑ CHECK YOUR UNDERSTANDING

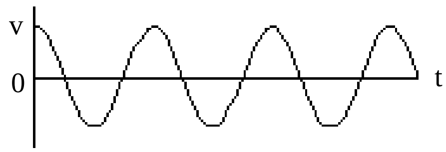
5. Which graph below is the x component of the *velocity*-versus-time graph for the object?



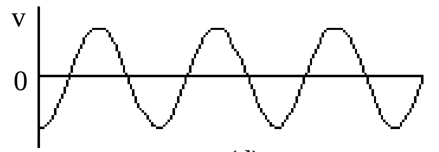
(a)



(b)



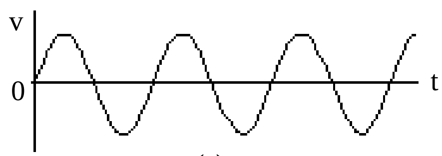
(c)



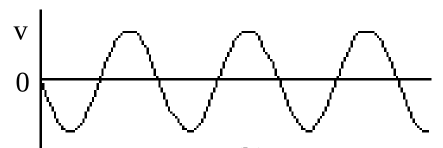
(d)

(e) None of the above.

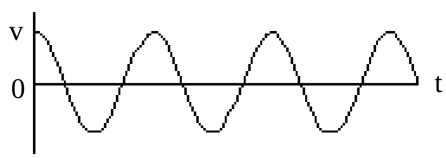
6. Which graph below is the y component of the *velocity*-versus-time graph for the object in question 5?



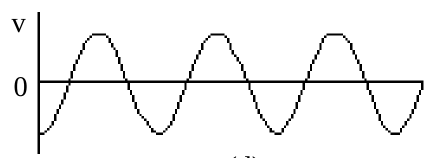
(a)



(b)



(c)



(d)

(e) None of the above.

☑ CHECK YOUR UNDERSTANDING

TA Name:____________________________

PHYSICS 1301 LABORATORY REPORT

Laboratory VI

Name and ID#: ____

Date performed: ______________ Day/Time section meets: ______________________

Lab Partners' Names: ______________________________________

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Problem # and Title: ______________________________________

Lab Instructor's Initials: ________

Grading Checklist	Points*
LABORATORY JOURNAL:	
PREDICTIONS (individual predictions and warm-up completed in journal before each lab session)	
LAB PROCEDURE (measurement plan recorded in journal, tables and graphs made in journal as data is collected, observations written in journal)	
PROBLEM REPORT:	
ORGANIZATION (clear and readable; logical progression from problem statement through conclusions; pictures provided where necessary; correct grammar and spelling; section headings provided; physics stated correctly)	
DATA AND DATA TABLES (clear and readable; units and assigned uncertainties clearly stated)	
RESULTS (results clearly indicated; correct, logical, and well-organized calculations with uncertainties indicated; scales, labels and uncertainties on graphs; physics stated correctly)	
CONCLUSIONS (comparison to prediction & theory discussed with physics stated correctly ; possible sources of uncertainties identified; attention called to experimental problems)	
TOTAL(incorrect or missing statement of physics will result in a maximum of 60% of the total points achieved; incorrect grammar or spelling will result in a maximum of 70% of the total points achieved)	
BONUS POINTS FOR TEAMWORK (as specified by course policy)	

* An "R" in the points column means to rewrite that section only and return it to your lab instructor within two days of the return of the report to you.