Appendix F: A Brief Introduction to RMS Measurements

A problem arises when one wishes to measure an alternating current or potential. All measuring instruments sample a signal over some period of time. A device that samples over a time longer than one period of the signal (such as the DMM) essentially measures the average signal. For sine or cosine functions, the average is zero, which doesn't tell you much about the signal strength.

The solution to this difficulty is to use root-mean-square (RMS) averaging. To eliminate the cancellation of the positive and negative parts of the sine function, it is squared, then the average is taken[1], and the square root of this average yields the RMS value.

For example, to find the RMS value of an AC current that has a maximum value of I_0:

$$I(t) = I_0 \sin(\omega t)$$
$$I^2(t) = I_0^2 \sin^2(\omega t)$$

$$\langle I^2 \rangle = \frac{1}{2\pi}\int_0^{2\pi} I_0^2 \sin^2(\omega t)\, d(\omega t)$$
$$= \frac{I_0^2}{2\pi}\int_0^{2\pi} \sin^2(\omega t)\, d(\omega t) = \frac{1}{2} I_0^2$$

$$I_{RMS} = \sqrt{\langle I^2 \rangle} = \frac{1}{\sqrt{2}} I_0$$

When in AC mode, your DMM displays the RMS values of current and voltage.

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1 When a quantity that varies with time is averaged, as in this case, the average value is often designated by putting angle brackets around the quantity. For example, the time average of a sinusoidally varying current is:

$$\langle I \rangle = \frac{I_0}{2\pi}\int_0^{2\pi} \sin(t)\, dt = 0$$