TA Orientation 2004
Activity #7 (2 points)

Solving Problems With Methods Questions

INDIVIDUAL TASK:

Solve your *assigned* problem (one of the two problems shown below).

Please include detailed diagram(s), define all variables, and show all steps in your solution. You can pick up and use a series of *Methods Questions* for the problem. These questions are a guide or framework for a logical, organized approach for solving the problem.

TIME: 20 minutes

PRODUCT:
Your problem solution. Your solution will **not** be graded for a correct answer. Instead, the solution will be graded for organization and logical progression.

QUANTUM MECHANICS PROBLEM. You are investigating the properties of very thin materials to study the behavior of quantum systems as their dimensionality goes from three to two. Your system is a surface held at liquid helium temperatures with dislocations in which electrons can be trapped. To predict the equipment needed to detect these trapped electrons, you first decide to calculate the first three energy levels assuming that the potential energy can be approximated as a two dimensional harmonic oscillator with a very small first order coupling between the two orthogonal dimensions. You decide to take the effect of that coupling as a perturbation to the pure harmonic oscillator potential energy. That perturbation is proportional to product of the distance of the electron from its equilibrium position in each dimension. You will work in units in which the mass of the electron is 1 and $\hbar = 1$.

Useful Mathematical Relationships:

$$\int_0^\infty dx * e^{-\alpha x} = \frac{1}{\alpha} \qquad \int_0^\infty dx * e^{-\alpha x^2} = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}} \qquad \int_0^\infty dx * x^n e^{-\alpha x} = \frac{\Gamma[n+1]}{\alpha^{n+1}}$$

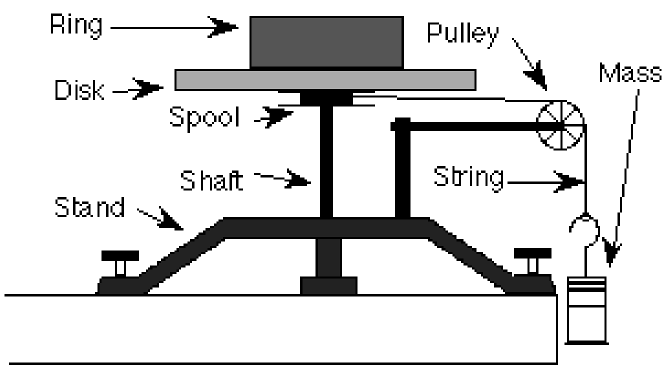
$$\int_0^\infty dx * x^m e^{-\alpha x^2} = \frac{\Gamma\left[\frac{m+1}{2}\right]}{2\alpha^{\frac{m+1}{2}}} \qquad \Gamma(n) = \frac{\Gamma(n+1)}{n} \qquad \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

Fundamental Concepts and Principles:

$$-\frac{\hbar^2}{2m}\nabla^2\Psi = i\hbar\frac{\partial}{\partial t}\Psi \qquad \int_{-\infty}^{+\infty}\Psi^*\Psi = 1 \qquad \Delta x \Delta p \geq \frac{\hbar}{2}$$

$$\int_{-\infty}^{+\infty}\Psi^* \hat{H}\, \Psi = E$$

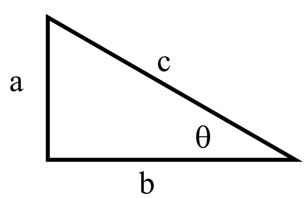
MOMENT OF INERTIA PROBLEM. While examining the engine of your friend's snow blower, you notice that the starter cord wraps around a cylindrical ring of metal. This ring is fastened to the top of a heavy, solid disk, "a flywheel", and that disk is attached to a shaft. You are intrigued by this configuration and wonder how to determine its moment of inertia. Your friend thinks that you just add up all of the individual moments of inertia of the parts to get the moment of inertia of the system. To test this idea you decide to build a laboratory model described below to determine the moment of inertia of a similar system from its motion. You think you can do it by just measuring the acceleration of the hanging weight, as shown in the diagram below.



A disk that is mounted on a sturdy stand by a metal shaft. Below the disk on the shaft is a metal spool to wind string around. A metal ring sits on the disk so both ring and disk share the same rotational axis. A length of string is wrapped around the spool and then passes over a pulley lined up with the edge of the spool. A weight is hung from the other end of the string so that the weight can fall past the edge of the table.

Calculate that the moment of inertia of the ring/disk/shaft/spool system as a function of the acceleration of the hanging weight and the radius of the spool.

Useful Mathematical Relationships:



For a right triangle: $\sin\theta = \frac{a}{c}$, $\cos\theta = \frac{b}{c}$, $\tan\theta = \frac{a}{b}$,

$a^2 + b^2 = c^2$, $\sin^2\theta + \cos^2\theta = 1$

Small angles: $\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{\theta^2}{2}$

For a circle: $C = 2\pi R$, $A = \pi R^2$

For a sphere: $A = 4\pi R^2$, $V = \frac{4}{3}\pi R^3$

If $Ax^2 + Bx + C = 0$, then $x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$\frac{d}{dz}\left(z^n\right) = nz^{n-1}$, $\frac{d}{dz}(\cos z) = -\sin z$, $\frac{d}{dz}(\sin z) = \cos z$, $\frac{df(z)}{dt} = \frac{df(z)}{dz}\frac{dz}{dt}$, $\int\left(\frac{dw}{dz}\right)dz = w$,

$\frac{d}{dz}\int w\,dz = w$, $\int z^n dz = \frac{z^{n+1}}{n+1}\ (n \neq -1)$

Fundamental Concepts and Principles:

$v_{x\,av} = \frac{\Delta x}{\Delta t}$	$s_{av} = \frac{dist}{\Delta t}$	$a_{x\,av} = \frac{\Delta v_x}{\Delta t}$	$\theta = \frac{\Delta C}{r}$	$\omega_{av} = \frac{\Delta\theta}{\Delta t}$	$\alpha_{av} = \frac{\Delta\omega}{\Delta t}$
$v_x = \frac{dx}{dt}$	$s = \frac{dr}{dt}$	$a_x = \frac{dv_x}{dt}$	$\omega = \frac{d\theta}{dt} = \frac{v_t}{r}$	$\alpha = \frac{d\omega}{dt} = \frac{a_t}{r}$	$\sum \vec{F} = m\vec{a}$
$\tau = rF_t$	$\sum \vec{\tau} = I\vec{\alpha}$	$W = \int_{path} \vec{F} \bullet d\vec{\ell}$	$KE = \frac{1}{2}mv^2$	$E_f - E_i = \Delta E_{transfer}$	$\vec{p} = m\vec{v}$
$\vec{p}_f - \vec{p}_i = \Delta\vec{p}_{transfer}$	$\vec{p}_{transfer} = \int \vec{F}dt$	$\vec{r}_{com} = \frac{\sum m_i \vec{r}_i}{\sum m_i} = \frac{\int \vec{r}dm}{\int dm}$		$I = \sum m_i r_i^2 = \int r^2 dm$	$\vec{L} = I\vec{\omega}$
$\vec{L}_f - \vec{L}_i = \Delta\vec{L}_{transfer}$	$\vec{L} = \vec{r} \times \vec{p}$	$\vec{L}_{transfer} = \int \vec{\tau}dt$		$f = \frac{1}{T}$	

Under Certain Conditions:

$x_f = \frac{1}{2}a(\Delta t)^2 + v_{ox}\Delta t + x_o$				$KE = \frac{1}{2}I\omega^2$		$PE = \frac{1}{2}kx^2$
$\theta_f = \frac{1}{2}\alpha(\Delta t)^2 + \omega_o \Delta t + \theta_o$	$a = \frac{v^2}{r}$					

$I_{cm}=fI_{ring}$: f (hollow sphere)=2/3; f (disk)=1/2; f(solid sphere)=2/5; f (solid rod, perpendicular to length)=1/3

Useful constants: 1 mile = 5280 ft, 1km = 5/8 mile, g = 9.8 m/s^2 = 32 ft/s^2, R_E = 4x10^3 miles, G = 6.7x10^{-11} Nm^2/kg^2