All problem-solving guides or frameworks *in any field* are:

- **based on expert-novice research;**
- **similar to on George Polya 's (1957) framework for mathematics problem solving.**

Physics problem-solving frameworks by different authors:

- **divide the framework into a different number of steps;**
- **Have different ways to say essentially the same thing;**
- **Emphasize different heuristics depending on the backgrounds of the students.**

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Polya (1957) Framework

1. Understand the Problem

Describe the problem:

- **Translate the situation and goals into the fundamental concepts of your field.**
- **Decide on the reasonable idealizations and approximations you need to make.**

2. Devise a Plan

Apply the specialized techniques (heuristics) of your field to develop a plan, *using the concepts of your field to connect the situation with the goal.*

Re-examine the description of the problem if a solution does not appear possible.

3. Carry Out the Plan

Follow your plan to the desired result.

Re-examine your plan if you cannot obtain the desired result.

4. Look Back

Determine how well your result agrees with your knowledge of similar behavior , within limits that you understand.

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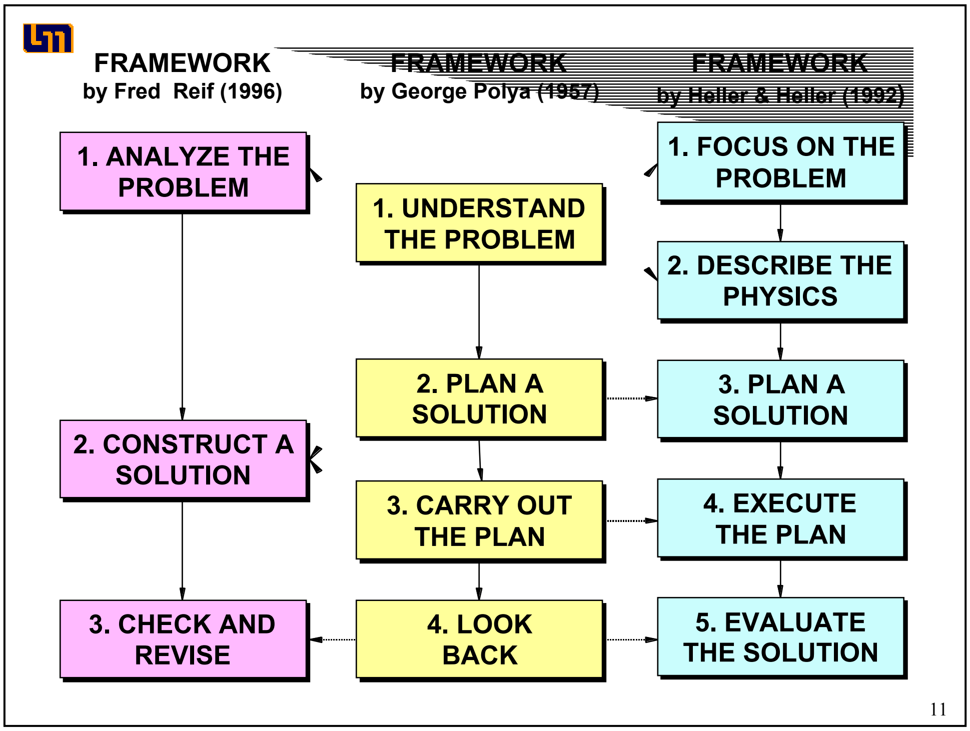

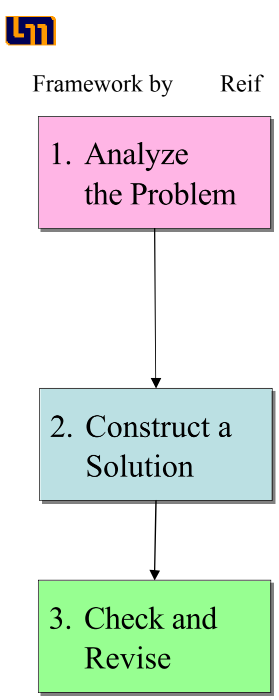

Basic Description: **draw diagram(s) to summarize situation; specify knowns and wanted (target) unknown(s) both symbolically and numerically.**

Refined Description: **Specify time sequence of events and identify time intervals where situation is different; use physics concepts to describe situation (e.g., velocity, acceleration, forces, etc.)**

Solve simplersubproblems *repeatedly*:

Examine status of problem for obstacle;
Select suitable sub-problem to overcome obstacle (e.g., apply basic relation)
Eliminate unwanted quantity

Check for Errors and Revise:

Goals attained? Well Specified? Self-consistent? Consistent with other known information? Optimal?

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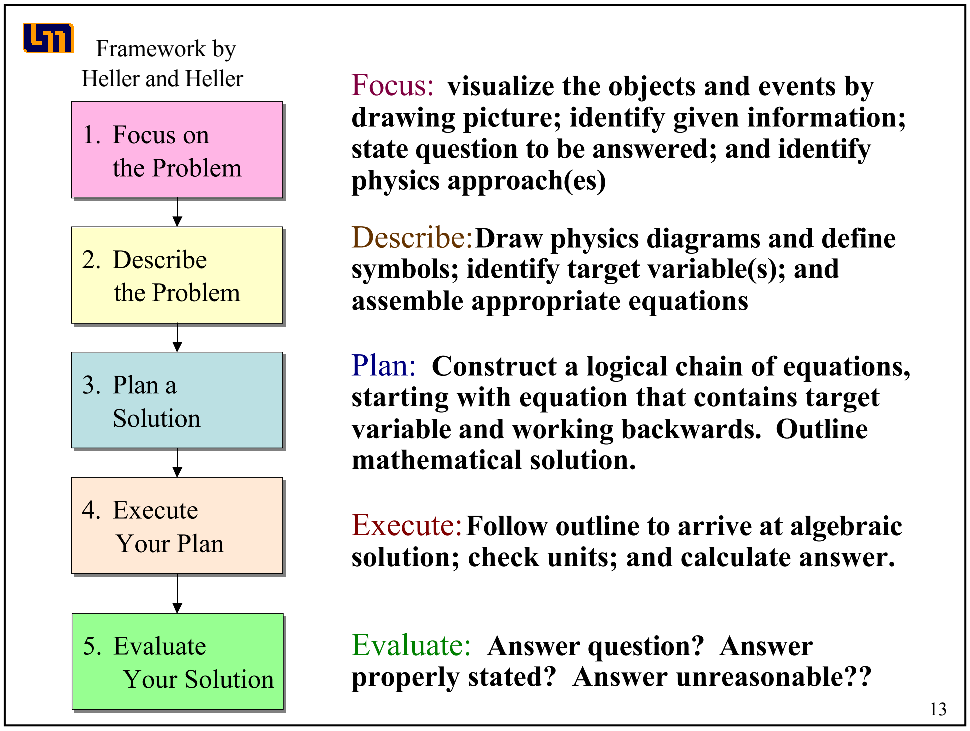
Framework by
Heller and Heller
1. Focus on the Problem
2. Describe the Problem
3. Plan a Solution
4. Execute Your Plan
5. Evaluate Your Solution
Focus: visualize the objects and events by drawing picture; identify given information; state question to be answered; and identify physics approach(es)
Describe: Draw physics diagrams and define symbols; identify target variable(s); and assemble appropriate equations
Plan: Construct a logical chain of equations, starting with equation that contains target variable and working backwards. Outline mathematical solution.
Execute: Follow outline to arrive at algebraic solution; check units; and calculate answer.
Evaluate: Answer question? Answer properly stated? Answer unreasonable??
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Activity #11b (1 point)

Lab Methods Questions and Problem Solving

GROUP TASKS:

You solved either a Quantum Mechanics Problem or a Moments of Inertia Problem in Activity 7.

1. Get in the same group as in Activity 7. Rotate Roles

If you solved the Quantum Mechanics Problem

2. I**ndividually** read through the problem-solving framework by Fred Reif (on the next page).
3. Go to the Answer Sheet on page 103. For each Method Question (Questions #1 - #10), identify and briefly describe the corresponding step and substep(s) from the *Understanding Basic Mechanics Framework* (Fred Reif).

If you solved the Moments of Inertia Problem:

2. **Individually** read through the problem-solving framework by Heller and Heller (on the page 100).
3. Go to the Answer Sheet on page 101. For each Method Question (Questions #1 - #8), identify and briefly describe the corresponding step and substep(s) from the *Competent Problem Solving Framework* (Heller & Heller).

TIME: 12 minutes.

One member from your group will be randomly selected to present your group's answers.

PRODUCT:

Answer Sheet for Activity #11b (Quantum Mechanics Problem), or
Answer Sheet for Activity #11b (Quantum Mechanics Problem)

Calculus-based Problem Solving Framework by Frederick Reif
(From *Understanding Basic Mechanics*, Wiley, 1995)

1. **Analyze the Problem:** Bring the problem into a form facilitating its subsequent solution.
   a. Basic Description -- clearly specify the problem by
      - describing the *situation*, summarizing by drawing diagram(s) accompanied by some words, and by introducing useful symbols; and
      - specifying compactly the *goal(s)* of the problem (wanted unknowns, symbolically or numerically)

   b. Refined Description -- analyze the problem further by
      - specifying the *time-sequence* of events (e.g., by visualizing the motion of objects as they might be observed in successive movie frames, and identifying the *time intervals* where the description of the situation is distinctly different (e.g., where acceleration of object is different); and
      - describing the situation in terms of important *physics concepts* (e.g., by specifying information about velocity, acceleration, forces, etc.).

2. **Construct a Solution:** Solve simpler subproblems repeatedly until the original problem has been solved.
   a. Choose subproblems by
      - examining the *status* of the problem at any stage by identifying the available known and unknown information, and the obstacles hindering a solution;
      - identifying available *options* for subproblems that can help overcome the obstacles; and
      - selecting a useful subproblem among these options.

   b. If the obstacle is lack of useful information, then apply a *basic relation* (from general physics knowledge, such as $m\mathbf{a} = \mathbf{F}_{TOT}$, $f_k = \square N$, $x = (1/2)a_x t^2$) to some *object or system* at some *time* (or between some times) along some *direction*.

   c. When an available useful relation contains an unwanted unknown, eliminate the unwanted quantity by combining two (or more) relations containing this quantity.

   Note: Keep track of wanted unknowns (underlined twice) and unwanted unknowns (underlined once).

3. **Check and Revise:** A solution is rarely free of errors and should be regarded as provisional until checked and appropriately revised.
   a. Goals Attained? Has all wanted information been found?

   b. Well-specified? Are answers expressed in terms of known quantities? Are units specified? Are both magnitudes and directions of vectors specified?

   c. Self-consistent? Are units in equations consistent? Are signs (or directions) on both sides of an equation consistent?

   d. Consistent with other known information? Are values sensible (e.g., consistent with known magnitudes)? Are answers consistent with special cases (e.g., with extreme or specially simple cases)? Are answers consistent with known dependence (e.g., with knowledge of how quantities increase or decrease)?

e. Optimal? Are answers and solution as clear and simple as possible? Is answer a general algebraic expression rather than a mere number?

Algebra-based Problem Solving Framework by P. Heller and K. Heller (From *The Competent Problem Solver.* Prentice Hall 1995)

1. Focus the Problem. Establish a clear mental image of the problem.

a. Visualize the situation and events by drawing a useful picture.
   - ♦ Show how the objects are related spatially and identify the time sequence of events, especially those times when an object experiences an abrupt change.
   - ♦ Identify the given information, in words and on the picture.

b. Precisely state the question to be answered.

c. Identify physics approach(es) that might be useful to reach a solution.
   - ♦ Which fundamental concepts of physics (e.g., kinematics, Newton's Laws, conservation of energy) might be useful for relating the physics to the problem situation.
   - ♦ List any approximations or constraints that are reasonable to apply to this situation.

2. Describe the Physics

a. Draw any necessary diagrams (e.g., motion diagrams, force diagrams, momentum diagrams) with coordinate systems that are consistent with the approach.
   - ♦ Define symbolically and consistently any quantities that are relevant to the situation.
   - ♦ Identify which of these quantities is known and which is unknown.

b. Identify the target variable(s) -- the quantity (or quantities) that will provide the answer to the question

c. Assemble the appropriate equations to quantify the physics concepts and constraints identified in your approach.

3. Plan a Solution

a. Construct a logical chain of equations, from those identified in the previous step, leading from the target quantity to quantities that are known.
   - ♦ Begin with the quantitative relationship that contains the *target variable*. Identify other unknowns in the equation.
   - ♦ Choose a new equation for one of these unknowns. Keep track of any additional unknowns.
   - ♦ Continue this process for each unknown.

b. Determine if this chain of equations is sufficient to solve for the target quantity by comparing the number of unknown quantities to the number of equations.

c. Outline the solution steps you will take to solve this chain of equations so that no algebraic loops are created. Work from the last equation to the first equation that contains the target quantity

4. Execute the Plan

Follow your solution outline

a. Arrive at an *algebraic* equation for your target quantity by following your chain of equations in reverse order (from quantities that are known to the target quantity).

b. Occasionally check the units of your process to help find mathematical errors.

c. Use numerical values to calculate the target quantity.

5. Evaluate the Answer

a. Does the mathematical result answer the question asked?

b. Is the result properly stated with appropriate units?

c. Is the result unreasonable?

Answer Sheet for Activity 11b (Moments of Inertia Problem)

Moment of Inertia Problem. For each Method Question below (Questions #1 - #8), identify the and briefly describe the corresponding step and sub-step(s) from the *Competent Problem Solver* (Heller & Heller) framework.

Step	Problem-solving Steps
1. Draw a side view of the equipment. Draw the velocity and acceleration vectors of the weight. Add the tangential velocity and tangential acceleration vectors of the outer edge of the spool. Also, show the angular acceleration of the spool. What is the relationship between the acceleration of the string and the acceleration of the weight if the string is taut? What is the relationship between the acceleration of the string and the tangential acceleration of the outer edge of the spool when the string is taut.	
2. Since you want to relate the moment of inertia of the system to the acceleration of the weight, you probably want to consider a dynamics approach (Newton's 2nd Law) especially using the torques exerted on the system. It is likely that the relationships between rotational and linear kinematics will also be involved.	
3. To use torques, first draw vectors representing all of the forces which could exert torques on the ring/disk/shaft/spool system. Identify the objects that exert those forces. Draw pictures of those objects as well showing the forces exerted on them.	
4. Draw a free-body diagram of the ring/disk/shaft/spool system. Show the locations of the forces acting on that systems. Label all the forces. Does this system accelerate? Is there an angular acceleration? Check to see you have all the forces on your diagram. Which of these forces can exert a torque on the system? Identify the distance from the axis of rotation to the point where each force is exerted on the system. Write down an equation which gives the torque in terms of the force that causes it. Write down Newton's second law in its rotational form for this system. Make sure that the moment of inertia includes everything in the	

Step	Problem-solving Steps
system.	
5. Use Newton's 3rd Law to relate the force of the string on the spool to the force of the spool on the string. Following the string to its other end, what force does the string exert on the weight? Make sure all the forces on the hanging weight are included in your drawing.	
6. Draw a free-body diagram of the hanging weight. Label all the forces acting on it. Does this system accelerate? Is there an angular acceleration? Check to see you have all the forces on your diagram. Write down Newton's second law for the hanging weight. How do you know that the force of the string on the hanging weight is not equal the weight of the hanging weight?	
7. Is there a relationship between the two kinematic quantities that have appeared so far: the angular acceleration of the ring/disk/shaft/spool system and the acceleration of the hanging weight? To decide, examine the accelerations that you labeled in your drawing of the equipment.	
8. Solve your equations for the moment of inertia of the ring/disk/shaft/spool system as a function of the mass of the hanging weight, the acceleration of the hanging weight, and the radius of the spool. Start with the equation containing the quantity you want to know, the moment of inertial of the ring/disk/shaft/spool system. Identify the unknowns in that equation and select equations for each of them from those you have collected. If those equations generate additional unknowns, search your collection for equations which contain them. Continue this process until all unknowns are accounted for. Now solve those equations for your target unknown.	

Answer Sheet for Activity 11b (Quantum Mechanics Problem)

Quantum Mechanics Problem. For each Method Question below (Questions #1 - #10), identify the and briefly describe the corresponding step and sub-step(s) from the *Understanding Basic Mechanics* (Fred Reif) framework.

Step	Problem-solving Steps
1. Write down the Hamiltonian for a one-dimensional harmonic oscillator in the x direction. What is the quantum mechanical operator that represents the kinetic energy? What quantum mechanical operator represents the potential energy?	
2. Solve the Schrodinger equation for this system. What function has the same form as its derivative? How do you determine if this equation has an eigenvalue?	
3. What is the normalized wave function for the ground state? Make sure all constants are determined. What is the eigenvalue for this wavefunction? How is the eigenvalue related to the ground state energy level?	
4. How do you construct the wave function for the first excited state from the ground state wave function? Determine the first excited state energy level.	
5. Write down the Hamiltonian for a two dimensional harmonic oscillator in the x direction and in the y direction where each dimension is independent. What is the Schrodinger equation for this system?	
6. Write down the functional form of the ground state wavefunction that is the solution to this Schrodinger equation. Why is this solution a product of a function of x only times a function of y only?	
7. Separate the two-dimensional harmonic oscillator equation into two one-dimensional equations and solve them. How do you assemble these solutions to construct the ground state solution and the ground state energy levels for the two dimensional equation?	

Step	Problem-solving Steps
8. How do you construct the wave functions for the first excited state in each dimension from the ground state wave function? Determine these energy levels. Are they the same or are they different? Why should this be true?	
9. Now add the coupling term to the two dimensional harmonic oscillator Hamiltonian as a perturbation to the potential energy. Write down the Hamiltonian of this system as the Hamiltonian to the pure two-dimensional harmonic oscillator plus a Hamiltonian that represents the perturbation of the harmonic oscillator potential energy. Calculate the effect of the perturbation Hamiltonian on the ground state energy of the pure harmonic oscillator in terms of the small parameter that gives the strength of the perturbation. In this calculation, you will encounter an integral over all space. Determine whether the integral is over an even function, an odd function, or a mixed function. In which of these cases is the value of the integral trivial?	
10. Calculate the effect of the perturbation Hamiltonian on the first exited state wavefunctions of the pure two-dimensional harmonic oscillator. How many different expectation values do you need to calculate? Use symmetry to determine which are non-zero. How do you use these perturbation energies to calculate the energy levels that you wish to determine?	