Teaching a Discussion Section

Page

I. Overview of Teaching a Discussion Session

The usual Cooperative Problem Solving (CPS) routine, like a game of chess, has three parts -- Opening Moves, a Middle Game, and an End Game. As in chess, both the opening moves and the end game are simple, and can be planned in detail. The middle game - collaborative problem solving -- has many possible variations.

Opening Moves (~ 5 minutes). Opening moves determine the mind set that students should have during the Middle Game -- the collaborative solving of a problem. The purpose of the opening moves is to answer the following questions for students.

- Why has this particular problem been chosen?
- What should we be practicing and learning while solving this problem?
- How much time will we have?
- What is the product we should have at the end of this time?

Educational research indicates that providing students this simple information before they start leads to better learning and higher achievement. An example of an opening move is shown in Figure1 on the next page.

Middle Game (~ 35 minutes). This is the learning activity -- students work collaboratively to solve the problem. During this time, your role is one of listener and facilitator. You circulate around the room, listening to what students in each group are saying and observing what the Checker/ Recorder is writing. You intervene when a group needs to be coached on an aspect of physics or is not functioning well. At the end of the allotted time, you have your groups draw and write on the board the parts of the solution that you specified in your opening moves.

End Game (~ 10 minutes). The end game determines the mind-set students have when they leave the class -- do they think they learned something or do they think it was it a waste of their time. The purpose of the end game is to help students answer the following questions.

- What have I learned that I didn't know before?
- What did other students learn?
- What should I concentrate on learning next?

Figure 1. Example of Group Practice Problem,, Opening Moves, and End-game Questions

Skateboard Problem

You are helping your friend prepare a skateboard exhibition. The idea is for your friend to take a running start and then jump onto a heavy duty 15-lb stationary skateboard. Your friend, on the skateboard, will glide in a straight line along a short, level section of track, then up a sloped concrete wall. The goal is to reach a height of at least 6 feet above the starting point before rolling back down the slope. The fastest your friend can run and safely jump on the skateboard is 20 feet/second. Can this program work as planned? Your friend weighs in at 125 lbs.

Example of Opening Moves

We have been studying the conservation principles in class -- the conservation of energy and the conservation of momentum. The problem you will solve today was selected to help you learn *when and how to apply these principles.*

You will have 35 minutes to work on the problem. At the end of that time, you will be asked to draw your diagrams and list the equations you used to solve the problem on the board.

Example End-game Questions

Look at the momentum vector diagrams on the board. How are they the same and how are they different?

Is there different physics represented in the diagrams, or the same physics?

Look at the diagrams for group #1 and #5. What is missing in these diagrams?

Does the order -- x direction first or y direction first -- make any difference to the final solution?

That is, a good end game helps students consolidate their ideas and produces discrepancies that stimulate further thinking and learning. Typically, the instructor gives students a few minutes to examine what each group produced, then leads a whole-class discussion of the results. Your role as the instructor is to *facilitate* the discussion, making sure students are actively engaged in consolidating their ideas. Examples of some end-game questions are shown in Figure 1 above.

II. Outline for Teaching a CPS Discussion Session

This outline, which is described in more detail in the following pages, could serve as your "lesson plan" for each discussion session you teach.

Preparation Checklist

- ❑ New Group/Role assignments (if necessary, on overhead or written on board)
- ❑ Photocopies of Problem & Useful Information *(one per person)* OR □□□□ of useful information to put on board
- ❑ Photocopies of Answer Sheet (optional) or blank sheets of paper *(one per group)*
- ❑ Photocopies of problem solution *(one per person)*
- ❑ Group Evaluation forms (optional one per group) and extra photocopies of Group Roles Sheet

	Instructor Actions	What the Students Do
Opening Moves ~3-5 min.	⓪ Be at the classroom early ① Introduce the problem by telling students: a) what they should learn from solving problem; b) the part of the solution you want groups to put on board ② Prepare students for group work by: a) showing group/role assignments and classroom seating map; b) passing out Problem & Useful Information and Answer Sheet.	• Students sitting and listening • Students move into their groups, and begin to read problem. • Checker/Recorder puts names on answer sheet.
Middle Game ~35 min.	③ Coach groups in problem solving by: a) monitoring (diagnosing) progress of all groups b) helping groups with the most need. ④ Prepare students for class discussion by: a) giving students a "five-minute warning" b) selecting one person from each group to put specified part of solution on the board. c) passing out Group Evaluation Sheet (optional)	• Solve the problem: - participate in group discussion, - work cooperatively, - check each other's ideas. • Finish work on problem • Write part of solution on board • Discuss their group effectiveness
End Game ~10 min.	⑤ Lead a class discussion focusing on what you wanted students to learn from solving the problem ⑥ Discuss group functioning (optional) ⑦ Pass out the problem solution as students walk out the door.	• Participate in class discussion

III. Detailed Advice for TAs about
General Discussion Section Lesson Plan

Opening Moves

Step ⓪. Be at the Classroom Early

The classroom will probably need some preparation, so it is best to go in and lock the door, leaving your early students outside. [The best time for informal talks with students is **after** the class or during your office hours.]

Check out the equipment you will need to use. If you are using the blackboard, you need time to write on the board (a) group assignments (if new) and roles; (b) the part of the solution you want groups to write on the board (optional, see below)

Step ①. State the Purpose of This CPS Session (~ 2 minutes)

Introduce the problem by telling students:

a) `What They Should Learn`. Tell your students why the group problem was selected and *what they should learn from solving the problem*. For example: "For the past few weeks we have been studying the conservation of energy and the conservation of momentum. The problem you will solve in your groups today was designed to help you think about the difference between the two conservation laws and when to apply a conservation law."

b) `The Part of the Solution You Want Groups to Put on the Board.` For example, for the skateboard problem: "After about 30 minutes, I will randomly select one person from each group to write two things on the board, first your conservation diagram(s) with defined symbols; and second a list of the specific equations that you need to solve the problem. [It is helpful to write this on a board, as shown below] Then we will discuss the features of a good diagrams that are useful for solving problems."

1. Conservation Diagram(s) & Defined Symbols.
2. List of specific equations needed to solve the problem

DO NOT have students write their mathematics solutions on the board. You can tell by a list of specific equations whether the students have the right equations to solve the problem. Students will see the detailed mathematics solution when you hand out the solution at the end of class.

Step ②. Prepare Students for Group Work (~ 1 minute)

a) Group Role Assignments. If students are working in the same groups, remind them to rotate roles. If you have assigned new groups, show students their group assignments and roles. Then tell your students to move the chairs for their group.

b) Pass Out Materials. Provide While the students are getting into their groups, *pass out the Problem/information Sheet* ***and*** *Answer Sheet (or blank pages)* to each group. As you do this, make sure all groups are seated according to your map -- facing each other, close together but with enough space between groups for you to easily observe and circulate between groups.

Middle Game (~ 30-35 minutes)

There are two instructor actions during the middle game: coaching students in problem solving, and preparing students for the whole class discussion. You will spend most of this time coaching groups.

Step ③. Coach Groups in Problem Solving (~ 25-30 minutes)

Below is a brief outline of coaching groups. For detailed suggestions for coaching and intervening techniques, see pages 25 - 33.

a) Diagnose initial difficulties with the problem or group functioning. Once the groups have settled into their task, spend about five minutes circulating and ***observing*** all groups. Try not to explain anything (except trivial clarification) until you have observed all groups at least once. This will allow you to determine if a whole-class intervention is necessary to clarify the task (e.g., "I noticed that very few groups are drawing conservation diagrams. Be sure to draw and label a diagram. . . .").

b) Monitor groups and intervene to coach when necessary. Establish a circulation pattern around the room. Stop and observe each group to see how easily they are solving the problem and how well they are working together. Don't spend a long time with any one group. Keep well back from students' line of sight so they don't focus on you. Make a mental note about which group needs the most help. Intervene and coach the group that needs the most help. If you spend a long time with this group, then circulate around the room again, noting which group needs the most help. Keep repeating the cycle of (a) circulate and diagnose, (b) intervene and coach the group that needs the most help.

Step ④. Prepare Students for Class Discussion (~ 5 minutes)

a) Five-minute Warning. About five minutes before you want students to stop, warn the class that they have only five minutes to wind up their solution. Then circulate around the class once more to determine the progress of the groups. Make a mental note of what you need to discuss with the class.

b) Posting Partial Group Solution. Tell one person in each group, who is *not* the Recorder/Checker, to write the (previously specified) part of their solution on the board (or butcher paper if there is not enough board space). In the beginning of the course, select students who are obviously interested and articulate. Later in the course, it is sometimes effective to occasionally select a student who has not participated in their group as much as you would like. This reinforces the fact that *all* group members need to know and be able to explain what their group did.

c) Pass out Group Functioning Evaluation form (optional). If you decided to have your groups evaluate their effectiveness, pass out the forms (one per group) and have groups complete the form.

End Game (~ 10 - 15 minutes)

The end-game discussion focuses on what you told students they would learn from solving this problem. The purpose is to help students consolidate their ideas and produce discrepancies that stimulate further thinking and learning.

After group pictures and equation list are posted (on board, whiteboards, butcher paper) for all to see, give students a few minutes to compare the results from each group. Then lead the class discussion

Step ⑤. Lead a Class Discussion (~ 10 minutes)

The whole-class discussion is always based on the groups, with individuals only acting as representatives of a group. This avoids putting one student "on the spot." The trick is to conduct a discussion about the problem solution without (a) **telling** the students the "right" answers or becoming the final "authority" for the right answers, and (b) without focusing on the "wrong" results of one group and making them feel stupid or resentful. To avoid these pitfalls, you could try starting with general, open-ended questions such as:

- ♦ How are the representations of the conservation of energy and conservation of momentum similar? [Need to consider initial and final states of the system, and whether there is a transfer into or out of the system]
- ♦ How are the representations different? [momentum is a vector; energy is not.]

In the beginning of a course, students naturally do not want to answer questions. They unconsciously play the “waiting game” -- if we wait long enough, instructors will answer their own questions and we won’t have to think. We recommend counting silently up to at least 30 after you have asked a question. Usually students get so uncomfortable with the silence that somebody speaks out. If not, call on a group by number: “Group 3, what do you think?”
After the general questions, you can become more specific. Of course, the specific question you ask will depend on what you observed while groups were solving the problem and what your groups write on the board. For the skateboard problem, some example questions might include:

- Which representations on the board include all the assumptions and information necessary to solve the problem? Why is this important?
- Do you think the assumptions you needed to make to solve this problem (no transfer of energy or momentum) made a big difference in your answer? Why or why not?

Remember to count silently up to 30, then call on a group if necessary. Always encourage an individual to get help from other group members if he or she is "stuck."

Encourage groups to talk to each other by redirecting the discussion back to the groups. For example, when a group reports their answer to a question, ask the rest of the class to comment: "What do the rest of you think about that?" This helps avoid the problem of you becoming the final "authority" for the right answer.

Step ⑥. Discuss group functioning (optional, ~ 5 minutes)

An occasional whole-class discussion of group functioning is essential. Students need to *hear* the difficulties other groups are having, *discuss* different ways to solve these difficulties, and receive *feedback* from you (see Chapter 7, page /31/). Randomly call on one member of from each group to report their group answer to the following question on the Evaluation form:

- one difficulty they encountered working together, or
- one way they could interact better next time.

After each answer, ask the class for additional suggestions about ways to handle the difficulties. Then add your own feedback from observing your groups (e.g., "I noticed that many groups are coming to an agreement too quickly, without considering all the possibilities. What might you do in your groups to avoid this?")

Step ⑦. Pass out the solution.

Passing out the solution is important to the students. They need to see good examples of solutions to improve their own problem solving skills. Again, it is important to pass them out as the last thing you do -- as students leave the room. If you pass them out earlier, your students will ignore anything that you say after you have passed them out.

IV. What are the Characteristics of a Good Group Problem?

Good group problems encourage students to use an organized, logical problem-solving framework instead of their novice, formula-driven, "plug-and-chug" strategy. Specifically, they should encourage students to (a) consider physics concepts in the context of real objects in the real world; (b) view problem-solving as a series of decisions; and (c) use their conceptual understanding of the fundamental concepts of physics to qualitatively analyze a problem *before* the mathematical manipulation of formulas.

In other words, good group problems place “barriers” on all solution paths that do not involve using a logical and organized problem-solving framework.

- ✓ It is difficult to use a formula to plug numbers to get an answer.
- ✓ It is difficult to find a solution pattern to match to get an answer. [A solution pattern is a memorized procedure for solving “inclined plane problems”, “free fall problems,” and son on.]
- ✓ It is difficult to solve the problem without first analyzing the problem situation.
- ✓ Physics words such as “inclined plane,” “starting from rest,” or “inelastic collision” are avoided as much as possible.

In addition, group problems should have an appropriate *level of difficulty* for its intended use (group practice problem or graded/test problem). All group problems should be more difficult to solve than easy problems typically given on an individual test. But the increased difficulty should be primarily conceptual, not mathematical. **Difficult mathematics is best accomplished by individuals, not by groups.** So problems that involve long, tedious mathematics but little physics, or problems that require the use of a shortcut or "trick" that only experts would be likely to know do not make good group problems. In fact, the best group problems involve the straight-forward application of the fundamental principles (e.g., the definition of velocity and acceleration, the independence of motion in the vertical and horizontal directions) rather than the repeated use of derived formulas (e.g., $v_f^2 - v_o^2 = 2ad$).

The application of some of these criteria to the Skateboard Problem (page 36) is shown on the next page.

Table 1. Criteria for A Good Group Problem

Criteria for Group Problem	Skateboard Problem
❶ **A group problem must be designed so that:**	
✓ There is something to discuss *initially* so that *everyone* (even the weakest member) can contribute to the discussion.	Students need to spend time initially drawing a picture of the situation.
✓ There are several decisions to make in solving the problem.	Students must decide what assumptions to make and what their target variable will be.
❷ **A group problem must be *challenging* enough so that**:	
✓ Even the best student in the group cannot immediately see how to solve the problem, and all students feel good about their role in arriving at a solution.	The problem cannot be solved by substitution of known values into momentum or energy equations.
✓ Knowledge of basic physics concepts is necessary to interpret the problem.	Students must apply the conservation of momentum and the conservation of energy.
✓ Students' alternative conceptions about the physics naturally arise and must be discussed.	Students must understand the *difference* between conservation of energy and momentum, and when it is appropriate to use these principles.
❸ **At the same time, the problem must be *simple* enough so that:**	
✓ The mathematics is not excessive or complex.	The problem requires only simple algebra.
✓ The solution path, once arrived at, can be understood, appreciated, and easily explained to all members of the group.	Once students have decided when and how to apply the conservation of momentum and energy, the solution is straightforward.
✓ A majority of groups can reach a solution in the time allotted.	Figuring out *how* to solve the problem, which takes the most time, can be done in the time allotted (about 35 minutes).

You may be asked (once a semester) to write a group practice problem. Don't start from scratch! Several faculty members at different colleges and universities have written context-rich problems for individual and cooperative groups. These problems are available at our web site: http://www.spa.umn.edu/groups/physed/

V. Decision Strategy for Judging Problems

Outlined below is a decision strategy to help you decide whether a problem is a good individual test problem, group practice problem, or group graded/test problem.

1. *Read* the problem statement. *Draw* the diagrams and *determine* the equations needed to solve the problem.

2. *Reject* if:
   ____ the problem can be solved in one step,
   ____ the problem involves long, tedious mathematics, but little physics; or
   ____ the problem can only be solved easily using a "trick" or shortcut that only experts would be likely to know. (In other words, the problem should be a straight-forward application of fundamental concepts and principles.)

3. *Check* for the twenty-one characteristics (see page 47-53) that make a problem more difficult:

Approach	Analysis	Mathematical Solution
1. Cues Lacking ___ A. No target variable ___ B. Unfamiliar context 2. Agility with Principles ___ A. Choice of principle ___ B. Two principles ___ C. Abstract principle 3. Non-Standard Application ___ A. Atypical situation ___ B. Unusual target	4. Excess or Missing Info. ___ A. Excess data ___ B. Numbers required ___ C. Assumptions 5. Seemingly Missing Info. ___ A. Vague statement ___ B. Special constraints ___ C. Diagrams 6. Additional Complexity ___ A. >2 subparts ___ B. 5+ terms ___ C. Vectors	7. Algebra required ___ A. No numbers ___ B. Unknown(s) cancel ___ C. Simultaneous eqns. 8. Targets Math Difficulty ___ A. Calc/vector algebra ___ B. Lengthy algebra

4. *Decide* if the problem would be a good group practice problem (20 - 25 minutes), a good group test problem (45 - 50 minutes), or a good (easy, medium, difficult) individual test problem, depending on three factors:
   (a) the complexity of mathematics,
   (b) the timing (when problem is to be given to students), and
   (c) the number of difficulty characteristics of the problem.

   Use the tables on the following page

Type of Problem	Timing	Diff. Ch.
Group Practice Problems should be shorter and mathematically easier than graded/test group problems.	• just introduced to concept(s) • just finished study of concept(s)	2 - 3 3 - 4
Graded/Test Group Problems can be more complex mathematically.	• just introduced to concept(s) • just finished study of concept(s)	3 - 4 4 - 5

Type of Problem	Timing	Diff. Ch.
Individual Problems can be easy, medium-difficult, or difficult: Easy	 • just introduced to concept(s) • just finished study of concept(s)	 0 -1 1 - 2
Medium-difficult	• just introduced to concept(s) • just finished study of concept(s)	1 - 2 2 - 3
Difficult	• just introduced to concept(s) • just finished study of concept(s)	2 - 3 3 - 4

There is considerable overlap in the criteria, so many problems can be judged both a good group practice or graded/test problem *and* a good easy, medium-difficult, or difficult individual problem.

VI. Difficulty Characteristics of Problems

There are twenty-one characteristics of a problem that can make it more difficult to solve than a standard textbook exercise. These difficulty characteristics are described below. Two problems are used to illustrate some of these difficulty characteristics. These problems and their partial solutions are shown in Figures 1 and 2.

Approach

The traits in the Approach category are grouped together because they all affect how a student decides which concepts, principles and laws to apply to a problem. In traditional textbook problem this is often given to the students either by a direct statement, such as "the carts have an inelastic collision" or merely by placing the problem at the end of the chapter under a subheading such as "Inelastic Collisions." Without such cues, the following 7 problem traits can make it more difficult for students to decide how to approach a problem.

1. Problem statement lacks standard cues

Novice problem solvers often decide on an approach from the "cues" in a problem statement. The two difficulty traits in this subcategory thwart this tendency.

A. `No explicit target variable.` The unknown variable of the problem is not explicitly stated. Problems with this difficulty trait typically include statements such as: "Will this plan (design) work?" or "Should you fight this traffic ticket in court."

B. `Unfamiliar context.` The context of the problem is *very* unfamiliar to the students (e.g., cosmology, molecules). The CO_2-ion problem (Figure 2, page 49) is an example of a classic Coulomb's Law problem that is more difficult to solve because of students' lack of familiarity with an abstract molecular context.

2. Solution Requires Agility in Using Principles

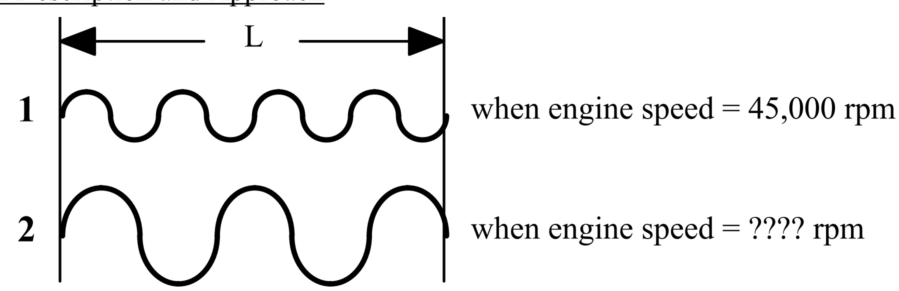
As novices in physics, most students are not initially very adept at using fundamental principles they just learned, or in making connections between principles. Problem statements that require the students to appreciate a concept's complexity will be more difficult for them. The next three difficulty traits are examples of how problem statements can force students into be fluent with principles.

A. `Choice of useful principles`. The problem has more than one possible set of useful concepts that could be applied for a correct solution. For example, consider a problem with a box sliding down a ramp. Typically either Newton's Laws of Motion or the conservation of energy will lead to a solution, but deciding which principles to use can be difficult for students.

Figure 1: Engine Vibration Problem and Partial Solution

You are working for an aerospace company on a team assigned to fix a design error in a new jet engine. It is important for the design that none of the engine parts vibrate too much during normal engine operation. This engine contains a control wire that is held at both ends. The wire develops a standing wave with 7 nodes (not counting the ends) at the engine turbine's maximum rotational speed of 45,000 revolutions per minute. The amplitude of this standing wave is not large enough to be a problem. A problem occurs, however, at a lower engine speed, where a larger amplitude standing wave with 4 nodes (not counting the ends) develops. You have been assigned to calculate the engine turbine's rotational speed when the 4-node standing wave occurs. The lead engineer tells you that the frequency of the standing wave on the wire is directly proportional to the turbine rotational speed. You also know that for any wave, its wavelength is related to its frequency and the speed of a wave on the wire.

Problem Description and Approach



Approach: Use the definition of the velocity of a wave.

Mathematical Expressions Needed for Solution:

From General Principle(s):	$v_1 = \lambda_1 f_1$	$v_2 = \lambda_2 f_2$	
From Problem Analysis:	$s_1 = \alpha f_1$	$s_2 = \alpha f_2$	(vague statement in problem)
	$4\lambda_1 = L$	$2.5\lambda_2 = L$	(from diagram)
	$v_1 = v_2$		(constraint)

7 equations and 8 unknowns: unknown L factors out of final solution

B. Two general principles. The correct solution requires students to use two or more major principles. Examples include pairings such as Newton's Laws and kinematics, conservation of energy and momentum, conservation of energy and kinematics, or linear kinematics and torque.

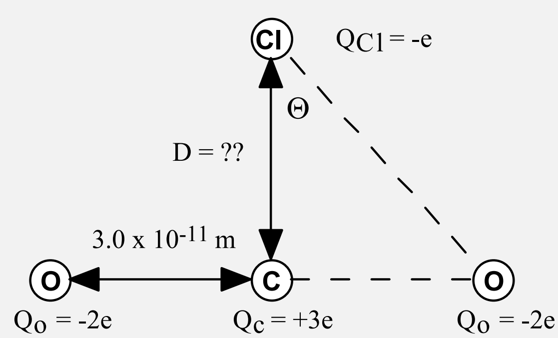
C. Very abstract principles. The central concept in the problem is an abstraction of another abstract concept. While concepts like angular momentum and electric fields are abstract, their presence alone does not warrant a difficulty trait. Rather, when the problem requires the students to use an abstract concept that is based upon another abstract concept, then this difficulty trait is present. Examples of very abstract principles include electric potential and magnetic flux.

Figure 2. C0₂-ion Problem and Partial Solution

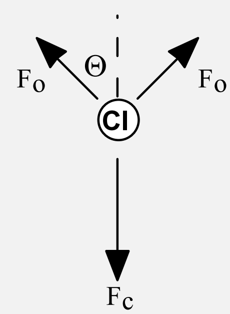
You are spending the summer working for a chemical company. Your boss has asked you to determine where a chlorine ion of effective charge -e would situate itself near carbon-dioxide ion. The carbon dioxide ion is composed of 2 oxygen ions each with an effective charge -2e and a carbon ion with an effective charge +3e. These ions are arranged in a line with the carbon ion sandwiched between the two oxygen ions. The distance between each oxygen ion and the carbon ion is 3.0 x 10^{-11} m. What is the equilibrium distance for the chlorine ion relative to the carbon ion? Assume that the chlorine ion is on a line which is perpendicular to the axis of the carbon dioxide ion, and that the line goes through the carbon ion. For simplicity, also assume that the carbon-dioxide ion does not deform in the presence of the chlorine ion.

Problem Analysis

SKETCH



FORCE DIAGRAM



Approach: Use Newton's Law of Motion and Coulomb's Law..

Mathematical Expressions Needed for Solution:

From General Principle(s): $\sum \vec{F} = 0$ so $2F_O \cos\theta = F_c$ (vector analysis)

$$F_O = \frac{kq_O q_{Cl}}{D^2 + a^2} \qquad F_C = \frac{kq_C q_{Cl}}{D^2}$$ (from diagram, Coulomb's Law)

From Problem Analysis: $\cos\theta = \dfrac{D}{\sqrt{D^2 + a^2}}$ (from diagram)

Mathematical Expressions Needed for Solution:

$$2F_{O,Y} = F_C$$

$$2\frac{kq_O q_{Cl}}{D^2 + a^2}\cos\theta = \frac{kq_C q_{Cl}}{D^2}$$

Solution requires detailed algebra:

3. Non-standard Application of Concepts and Principles

Students typically learn new concepts or principles by solving problems that require only a simple, straightforward application of the concept or principle. For example, students initially learn Coulomb's Law by solving problems that require students to find the total force on a charge located at known distances from other charges. The two difficulty traits in this subcategory require students to generalize their problem-solving knowledge to atypical situations or combinations beyond the standard situations.

A. Atypical situation. The setting, constraints, or complexity is unusual compared with textbook problems. That is, the problem combines objects or interactions that are not normally put together. The CO_2-ion problem (Figure 2, page 49) has this trait since the electric field the chloride ion encounters is from three charges in a line instead of the usual dipole. Another atypical situation might include an energy conservation problem involving multiple potential energy terms in the total energy.

B. Unusual target variable. The problem involves an atypical target variable when compared with homework problems. The CO_2-ion problem (Figure 2, page 49) has this trait since the students must solve for the equilibrium distance, a distance that is usually supplied in standard problems.

Analysis of Problem

Problem analysis is the translation of the written problem statement into a complete physics description of the problem. It includes a determination of which physics concepts apply to which objects or time intervals, specification of coordinate axes, physics diagrams (e.g., a vector momentum diagram), specification of variables (including subscripts), and the determination of special conditions, constraints, and boundary conditions (e.g., $a_1 = a_2 =$ constant). The next 9 traits are all examples of how problems that require a careful and complete qualitative analysis are more difficult for students to solve.

4. Excess or Missing Information

Typical textbook problems give exactly the information necessary to solve the problem. Consequently, some students use these values in helping them decide which "formulas" they need to solve the problem. Excess or missing information in a problem thwarts this naive strategy and requires students to analyze the problem situation to decide how to proceed

A. Excess numerical data. The problem statement includes more data than is needed to solve the problem. For example, the inclusion of both the static and kinetic coefficients of friction in a problem requires students to decide which frictional force is applicable to the situation.

B. `Numbers must be supplied.` The problem requires students to either remember a common number, such as the boiling temperature of water, or to estimate a number, such as the height of a woman.

C. `Uncommon assumptions.` The problem requires students to generate an *uncommon* simplifying assumption to eliminate an unknown variable. *All* problems require students to use their common sense knowledge of how the world works (e.g., boats move through water and not through the air!). Typically, assumptions, such as frictionless surfaces or massless strings, are explicitly made for the students in class or in textbooks. Therefore, asking students to make their own simplifying assumptions is a new and difficult task. Problems that require students to make their own simplifying assumptions are more difficult to solve. To be included as a difficulty trait, the simplifying assumption must be *uncommon*, such as ignoring a small frictional effect when it is not obvious to do so. The two categories of uncommon simplifying are neglect and ignore. The first category includes situations where the students must *neglect* a quantity, such as neglecting the mass of a flea when compared to the mass of a dog. The next category of assumptions involves *ignoring* effects that cannot be easily expressed mathematically, such as how a yo-yo's string changes its moment of inertia.

5. Seemingly Missing Information

The problem requires students to generate a mathematical expression from their analysis of the problem. This expression might be derived from their understanding of how real system work or from a careful diagram. The creativity involved in overcoming this class of obstacles is not normally encountered in textbook problems nor is it usually taught. There are three difficulty traits in this subcategory.

A. `Vague statement.` The problem statement introduces a vague, new mathematical statement. For example, if the problem statement tells the students that "A is proportional to B," then the students must not only translate the written statement into a mathematical expression, but then know where and how to use it. The engine-vibration problem has this difficulty trait (see Figure 1, page 48). Additional examples of statements that require a translation into a mathematical expression are: "The car has a weight of 1400 pounds, and 75% of that weight is carried by the front tires," and "The counterweight is always twice the mass of the package on the ramp."

B. `Special conditions or constraints.` The problem requires students to generate information from their analysis of the conditions or constraints. The engine-vibration problem (Figure 1, page 48) has this difficulty trait. Students must recognize from their analysis of this problem that the traveling wave velocities in the two resonance situations are equal: $v_1 = v_2$. Another example is the generation of the

relationship
$a_1 = a_2$ for the two masses in an ideal Atwood machine.

C. `Diagrams.` The problem requires students to extract information from a spatial diagram. The engine-vibration problem (Figure 1, page 48) has this difficulty trait. Students must use a diagram to relate the wavelength of a standing wave to the overall length of the wire by counting nodes. The CO_2-ion problem (Figure 2, page 49) also has this difficulty trait. Students must express the cosine of an angle between forces in terms of known and unknown distances.

6. Additional Complexity

The problems in this subcategory require students to be especially careful in their analysis and variable definitions. The more "pieces" students have to keep track of, the more difficult the problem.

A. `More than two subparts.` The problem solution requires students decompose the problem into more than two subparts. Two or more sub-parts can arise because there are more than two interacting objects or more than two important time intervals. Changing systems of interest for students can be hard. Also, novice problem-solvers will often lose sight of the problem goal through numerous subparts. Examples of this trait include such classic problems as the ballistic pendulum (which requires conserving energy before and after the impact, but not during the impact) or the massive-pulley Atwood machine (which requires analyzing the suspended masses and the pulley).

B. `Five or more terms per equation.` The problem involves five or more terms in a principle equation. Typical examples are problems in which 5 or more forces are acting on a single object along one axis, or there are 5 or more energy terms in the conservation of energy equation or finding the potential from 5 individual charges. Problem statements with this trait require special care in specifying variables and signs for each term

C. `Two directions (vector components).` The problem requires students to treat principles (e.g., forces, momentum) as vectors. This requires both the decomposition of the physics principle and the careful subscripting of variables. Some students are still tripped up taking vector components even after weeks of using vectors. For example, decomposing electric field vectors into components is one of the stumbling blocks in integrating the field of continuous charge distributions. The CO_2-ion problem (Figure 2, page 49) also demonstrates this difficulty trait. The solution requires the students to decompose the contribution to the net force from the oxygen atoms.

Mathematical Solution

Mathematical difficulty is last category of traits. A teacher can put into a problem some simple mathematical hurdles that prevent some students from reaching a final answer. Some of these are included in the last five traits.

7. Algebra Required

A strictly algebraic solution is challenging for many novice problem-solvers. Three problem types can require algebraic solutions.

A. `No numbers.` The problem statement does not use any numbers. Many students use numbers as placeholders to help them remember which variables are known and which are unknown. Therefore, if a problem is written without numbers, it is more difficult for the students.

B. `Unknown(s) cancel.` Problems are more difficult to solve when an unknown variable, such as a mass, ultimately factors out of the final solution. The students must not only decide how to solve the problem without all the cues they expect, but keep symbolic track of all the variables. The engine-vibration problem (Figure 1, page 48) has this trait: there are 7 equations and 9 unknowns, but the length of the wire and the proportionality constant factor out of the solution.

C. `Simultaneous equations.` The solution requires solving simultaneous equations. Simultaneous equations are hard for the students not only because of the algebra involved, but because there are at least two unknowns in each equation and they need to keep track of these variables. A typical circuit-analysis problem best illustrates this trait.

8. Targets Math Difficulties

The problems in this subcategory require students to use mathematics that is known to be problematic.

A. `Calculus or vector algebra.` The solution requires the students to use sophisticated vector algebra, such as cross products, or calculus. Most students are still learning these skills in their math courses and have not learned how to transfer these skills from their math class to their physics class.

B. `Lengthy or Detailed Algebra.` A successful solution to the problem is not possible without working through lengthy or detailed algebra. While these calculations are typically not difficult, they require careful execution. A typical example is a problem that requires students to solve a quadratic equation. The CO_2-ion problem (Figure 2, page 48) has this difficulty trait. This problem requires the students to correctly solve for the unknown that is part of a summation under a square root that is in the denominator.

VII. Examples of How to Judge Problems

Example 1. Engine Vibration Problem

1. *Read* the problem statement. *Draw* the diagrams and *determine* the equations needed to solve the problem.

   see Figure 1, page 48

2. *Reject* if:

   _No_ the problem can be solved in one step,

   _No_ the problem involves long, tedious mathematics, but little physics; or

   _No_ the problem can only be solved easily using a "trick" or shortcut that only experts would be likely to know. (In other words, the problem should be a straight-forward application of fundamental concepts and principles.)

3. *Check* for the twenty-one characteristics (see page 47-53) that make a problem difficult:

Approach	Analysis	Mathematical Solution
1. Cues Lacking	4. Excess or Missing Info.	7. Algebra required
___ A. No target variable	✓? A. Excess data	___ A. No numbers
___ B. Unfamiliar context	___ B. Numbers required	✓ B. Unknown(s) cancel
	___ C. Assumptions	___ C. Simultaneous eqns.
2. Agility with Principles		
___ A. Choice of principle	5. Seemingly Missing Info.	8. Targets Math Difficulty
___ B. Two principles	✓ A. Vague statement	___ A. Calc/vector algebra
___ C. Abstract principle	✓ B. Special constraints	___ B. Lengthy algebra
	✓ C. Diagrams	
3. Non-Standard Application		**Total = 5.5**
✓ A. Atypical situation	6. Additional Complexity	
___ B. Unusual target	___ A. >2 subparts	
	___ B. 5+ terms	
	___ C. Vectors	

4. *Decide* using Tables on page 46. This problem has a difficulty rating of 5 - 6, one in the approach, 4 in the analysis, and one in the mathematical solution. The mathematics involved is easy. This makes the problem too difficult for even a graded/test group problem! The problem should be rewritten to eliminate at least 1 of the difficulty characteristics in the analysis.

Example 2. CO_2-ion Problem

1. *Read* the problem statement. *Draw* the diagrams and *determine* the equations needed to solve the problem.

   See Figure 2 on page 49.

2. *Reject* if:

   _No_ the problem can be solved in one step,

   _No_ the problem involves long, tedious mathematics, but little physics; or

   _No_ the problem can only be solved easily using a "trick" or shortcut that only experts would be likely to know. (In other words, the problem should be a straight-forward application of fundamental concepts and principles.)

3. *Check* for the twenty-one characteristics (see page 47-53) that make a problem difficult:

Approach	Analysis	Mathematical Solution
1. Cues Lacking ___ A. No target variable ✓ B. Unfamiliar context 2. Agility with Principles ___ A. Choice of principle ___ B. Two principles ___ C. Abstract principle 3. Non-Standard Application ✓ A. Atypical situation ✓ B. Unusual target	4. Excess or Missing Info. ___ A. Excess data ✓? B. Numbers required ___ C. Assumptions 5. Seemingly Missing Info. ___ A. Vague statement ___ B. Special constraints ✓ C. Diagrams 6. Additional Complexity ___ A. >2 subparts ___ B. 5+ terms ✓ C. Vectors	7. Algebra required ___ A. No numbers ___ B. Unknown(s) cancel ___ C. Simultaneous eqns. 8. Targets Math Difficulty ___ A. Calc/vector algebra ✓ B. Lengthy math **Total = 6.5**

4. *Decide* using Tables on page 46. This problem has a difficulty rating of 6 - 7, three in the approach, 3 in the analysis, and one in the mathematical solution. The mathematics involved is easy, but long. This makes the problem too difficult for even a graded/test group problem! The problem should be rewritten to eliminate at least 2 of the difficulty characteristics.

Example 3. Skateboard Problem

You are helping your friend prepare for her next skate board exhibition. For her program, she plans to take a running start and then jump onto her heavy-duty 15-lb stationary skateboard. She and the skateboard will glide in a straight line along a short, level section of track, then up a sloped concrete wall. She wants to reach a height of at least 10 feet above where she started before she turns to come back down the slope. She has measured her maximum running speed to safely jump on the skateboard at 7 feet/second. She knows you have taken physics, so she wants you to determine if she can carry out her program as planned. She tells you that she weighs 100 lbs.

Assume that students have just started to study the conservation of energy and momentum.

1. *Read* the problem statement. *Draw* the diagrams and *determine* the equations needed to solve the problem.

2. *Reject* if:

   _No_ the problem can be solved in one step,

   _No_ the problem involves long, tedious mathematics, but little physics; or

   _No_ the problem can only be solved easily using a "trick" or shortcut that only experts would be likely to know. (In other words, the problem should be a straight-forward application of fundamental concepts and principles.)

3. *Check* for the twenty-one characteristics see page 47-53) that make a problem difficult:

Approach	Analysis	Mathematical Solution
1. Cues Lacking _√_ A. No target variable ___ B. Unfamiliar context 2. Agility with Principles ___ A. Choice of principle _√_ B. Two principles ___ C. Abstract principle 3. Non-Standard Application ___ A. Atypical situation ___ B. Unusual target	4. Excess or Missing Info. ___ A. Excess data ___ B. Numbers required _√_ C. Assumptions 5. Seemingly Missing Info. ___ A. Vague statement ___ B. Special constraints ___ C. Diagrams 6. Additional Complexity ___ A. >2 subparts ___ B. 5+ terms ___ C. Vectors	7. Algebra required ___ A. No numbers ___ B. Unknown(s) cancel ___ C. Simultaneous eqns. 8. Targets Math Difficulty ___ A. Calc/vector algebra ___ B. Lengthy algebra **Total = 3**

4. *Decide* using Tables on page 46. This problem has a difficulty rating of 3, two of which are in the approach. The mathematics involved is easy. This would make a decent group practice problem or a medium-difficult individual test problem. It is too easy for a graded/test group problem. If you were teaching this as a group practice problem, you could expect students to spend more time on the setup of the problem, and less time on the math.

Example 4. Electric and Gravitational Force

Electric and Gravitational Force: You and a friend are reading a newspaper article about nuclear fusion energy generation in stars. The article describes the helium nucleus, made up of two protons and two neutrons, as very stable so it doesn't decay. You immediately realize that you don't understand why the helium nucleus is stable. You know that the proton has the same charge as the electron except that the proton charge is positive. Neutrons you know are neutral. Why, you ask your friend, don't the protons simply repel each other causing the helium nucleus to fly apart? Your friend says she knows why the helium nucleus does not just fly apart. The gravitational force keeps it together, she says. Her model is that the two neutrons sit in the center of the nucleus and gravitationally attract the two protons. Since the protons have the same charge, they are always as far apart as possible on opposite sides of the neutrons. What mass would the neutron have if this model of the helium nucleus works? Is that a reasonable mass? Looking in your physics book, you find that the mass of a neutron is about the same as the mass of a proton and that the diameter of a helium nucleus is 3.0×10^{-13} cm.

Assume that students have just finished studying electric forces.

1. *Read* the problem statement. *Draw* the diagrams and *determine* the equations needed to solve the problem.

2. *Reject* if:

   _No_ the problem can be solved in one step,

   _No_ the problem involves long, tedious mathematics, but little physics; or

   _No_ the problem can only be solved easily using a "trick" or shortcut that only experts would be likely to know. (In other words, the problem should be a straight-forward application of fundamental concepts and principles.)

3. *Check* for the twenty-one characteristics (see page 47-53) that make a problem difficult:

Approach	Analysis	Mathematical Solution
1. Cues Lacking _√_ A. No target variable _√_ B. Unfamiliar context 2. Agility with Principles ___ A. Choice of principle _√_ B. Two principles ___ C. Abstract principle 3. Non-Standard Application _√_ A. Atypical situation ___ B. Unusual target	4. Excess or Missing Info. ___ A. Excess data ___ B. Numbers required ___ C. Assumptions 5. Seemingly Missing Info. ___ A. Vague statement ___ B. Special constraints _√_ C. Diagrams 6. Additional Complexity ___ A. >2 subparts ___ B. 5+ terms ___ C. Vectors	7. Algebra required ___ A. No numbers ___ B. Unknown(s) cancel ___ C. Simultaneous eqns. 8. Targets Math Difficulty ___ A. Calc/vector algebra ___ B. Lengthy algebra

4. *Decide* using Tables on page 46. This has a difficulty rating of 5, four of which are in the approach. The mathematics involved is easy, but the difficulty is all in the setup. Students probably have not studied gravitational force lately, which makes the problem more difficult. This would be a difficult group test problem.

   If you were teaching this as a graded/test group problem, you could expect groups to spend most of their time on the setup of the problem, so don't worry if they haven't gotten to the math by the middle of the hour.