Facilitating an End Discussion (complete student solutions)

INDIVIDUAL TASKS:

**If you have already done the individual task from Activity #6, refer to your preparation notes from that activity.

On the following page is an introductory physics problem – pretend that your teaching team has decided to use this problem in the next discussion session.

1. Solve this problem by yourself.
2. Write down some notes about how you would prepare for this discussion session. Use the Discussion Preparation sheet as a guide.
   a. What is the learning focus for this problem that you will emphasize?
   b. What do you expect students to have difficulty with?
   c. What questions can you ask students?
3. Write up a detailed "solution" to this problem that you would hand out to your students at the end of class.

INDIVIDUAL & GROUP TASKS:

Following the problem statement are 8 complete student solutions to the problem. Notice that these are the same student solutions from Activity #6, but they are now longer. For this activity, you should pretend that you are approaching the end of teaching a discussion session with this problem. As you circulate the room one last time, you observe what students have written on their papers.

Choose 4 of the following 8 solutions to represent what your student groups have come to a consensus about for the problems. Ignore the other 4 solutions.

1. Based on the 4 completed solutions you have chosen, what will you ask student groups to put on the board for an end discussion?
2. After they put this on the board, what questions will you ask during the end-of-class discussion with all groups?

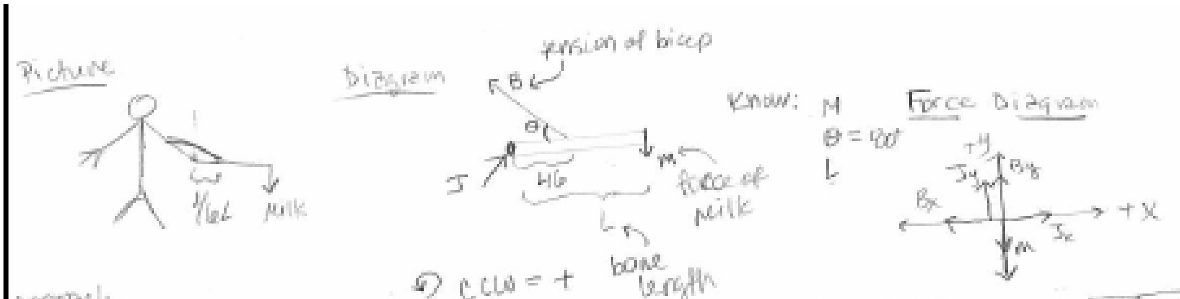
Be prepared to share your responses to these questions with your peers during TA Orientation.

NOTE: These partial student solutions were actually taken from individual solutions to a 1201 final exam problem in Fall 2005, from two different lecture sections. The problem was chosen because it is similar to most group problems given in discussion sessions.

Problem:

Your task is to design an artificial joint to replace arthritic elbow joints in patients. After healing, the patient should be able to hold at least a gallon of milk (3.76 liters) while the lower arm is horizontal. The bicep muscle is attached to the bone at the distance 1/6 of the bone length from the elbow joint, and makes an angle of 80^0 with the horizontal bone. For how strong of a force should you design the artificial joint? (The weight of the bone is negligible.)

STUDENT #1:



⟲ ccw = +

Approach

use forces $\Sigma F = 0$

use torques $\Sigma \tau = 0$

ignore bone weight

By Newton's 3rd Law the force of the joint on the bone is equal to the force of the bone on the joint.

$B_x = B\cos\theta$ $\quad J = \sqrt{J_x^2 + J_y^2}$

$B_y = B\sin\theta$

$\tan\theta = B_y/B_x$

$\Sigma F_x = J_x - B_x = 0$

$\Sigma F_y = J_y + B_y - M = 0$

$\Sigma \tau = B_y \frac{L}{6} - ML = 0$ or $B\sin\theta = \frac{L}{6} - ML$

Target: J

		unknown
Find J	$J = \sqrt{J_x^2 + J_y^2}$ ①	J_y, J_x
Find J_y	$J_y + B_y - M = 0$ ②	B_y
Find J_x	$J_x - B_x = 0$ ③	B_x
Find B_y	$B_y = B\sin\theta$ ④	
Find B_x	$B_x = B\cos\theta$ ⑤	

5 unknowns 5 equations

Put ⑤ into ③ and solve for J_x

$B_x = B\cos\theta$, $J_x = B\cos\theta$

put ④ into ② and solve for J_y

$B_y = \sin\theta$, $J_y = -\sin\theta + M$

now put J_y and J_x equations into ①

$J = \sqrt{(B\cos\theta)^2 + (-\sin\theta + M)^2}$

→ note that $B = \dfrac{\frac{L}{6} - ML}{\sin\theta}$

(from the equation for $\Sigma\tau$)

putting this into the equation for J:

$$J = \sqrt{\left(\left(\frac{\frac{L}{6} - ML}{\sin\theta}\right)\cos\theta\right)^2 + \left(-\sin\theta + M\right)^2}$$

We know that $\theta = 80°$ and the M is a gallon of milk but not its weight or mass (gallon is a unit of volume) so putting in θ gives:

$$J = \sqrt{\left(\left(\frac{\frac{L}{6} - ML}{\sin(80)}\right)\cos(80)\right)^2 + \left(-\sin 80 + M\right)^2}$$

check units:

$J = \sqrt{([\text{length}] - [\text{length}][\text{force}])^2 + [\text{force}]^2}$

$J = \sqrt{(\text{force})^2 + (\text{force})^2}$

J = force. OK units check

STUDENT #2:

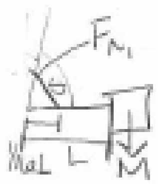
KNOWS:

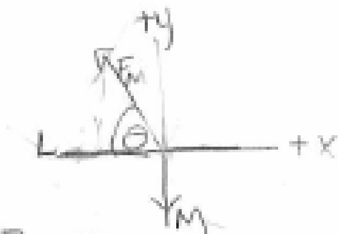
M - 3.76 LITERS.

θ - 80°

L2 = 1/16 L1

WEIGHT OF BONE NEGLIGIBLE.





Fm = FORCE OF MUSCLE

APPROACH:

Q. HOW STRONG A FORCE SHOULD THE ARTIFICAL JOINT BE MADE

USE FORCES NEGLECT BONE MASS

$\sum_x = L_x\cos\theta - M_x = 0$ $\sum = F_{m_y}\sin\theta = M_y - 0$

$\tau = rF_\perp$ $\sum\tau = F\cos\theta - m = 0$

FORCE OF MUSCLE $F_{m_x} = M_x - L_x$

FIND Fm

$F_m = M_x - L_x$

$L_x\cos\theta - m_x$

$M_x = L_x\cos\theta$

Fm

m_x

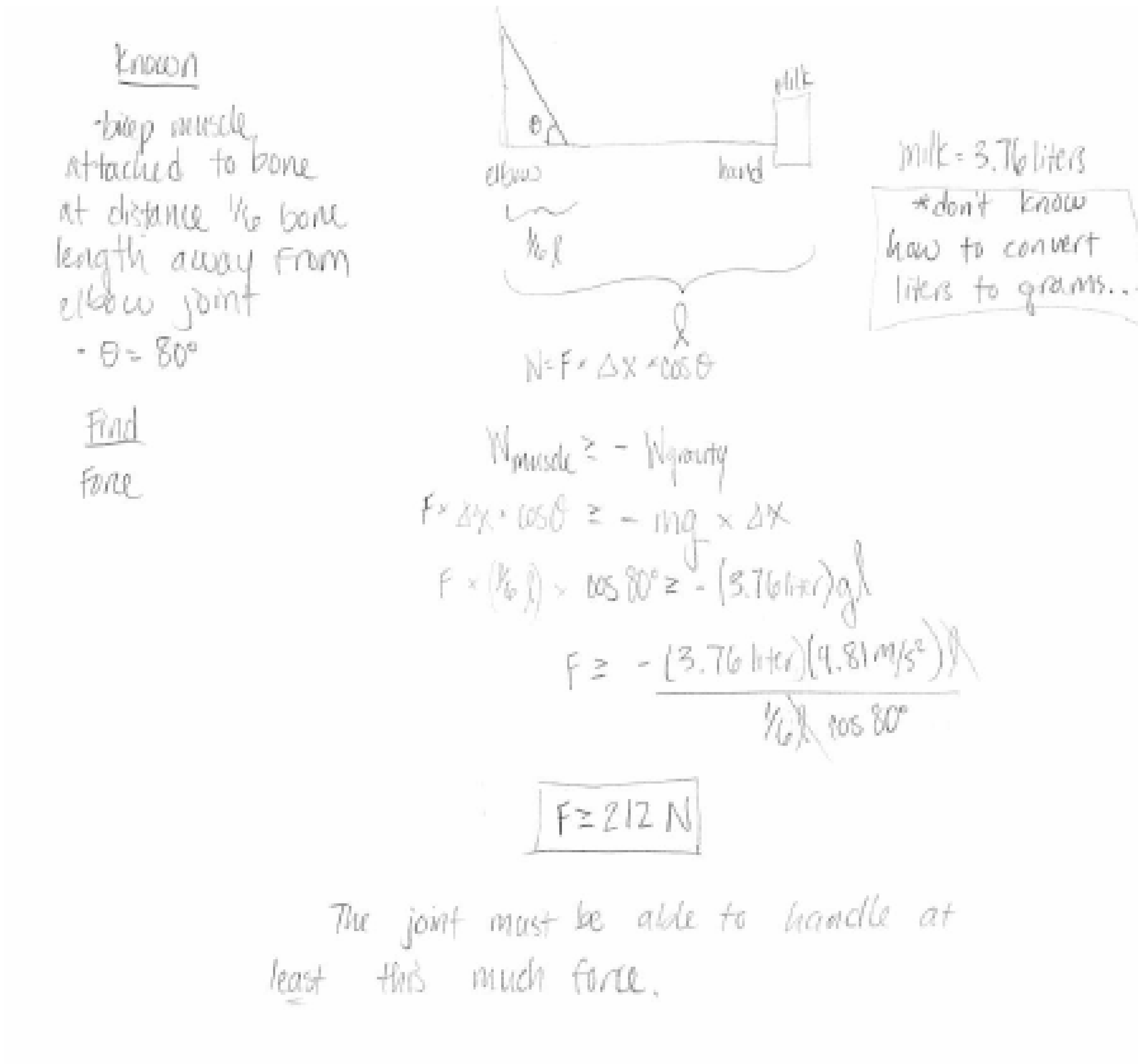
L_x

$L_x\cos\theta - F_{m_y}\sin\theta - F\cos\theta$

$L_x\cos\theta - F\cos\theta = F_{m_y}\sin\theta$

$$F_m = \frac{L_x\cos\theta - F\cos\theta}{\sin\theta}$$

STUDENT #3:



Known

- bicep muscle attached to bone at distance 1/6 bone length away from elbow joint
- $\theta = 80°$

Find

Force

milk = 3.76 liters

*don't know how to convert liters to grams...

$W = F \times \Delta x \times \cos\theta$

$W_{muscle} \geq - W_{gravity}$

$F \times \Delta x \times \cos\theta \geq - mg \times \Delta x$

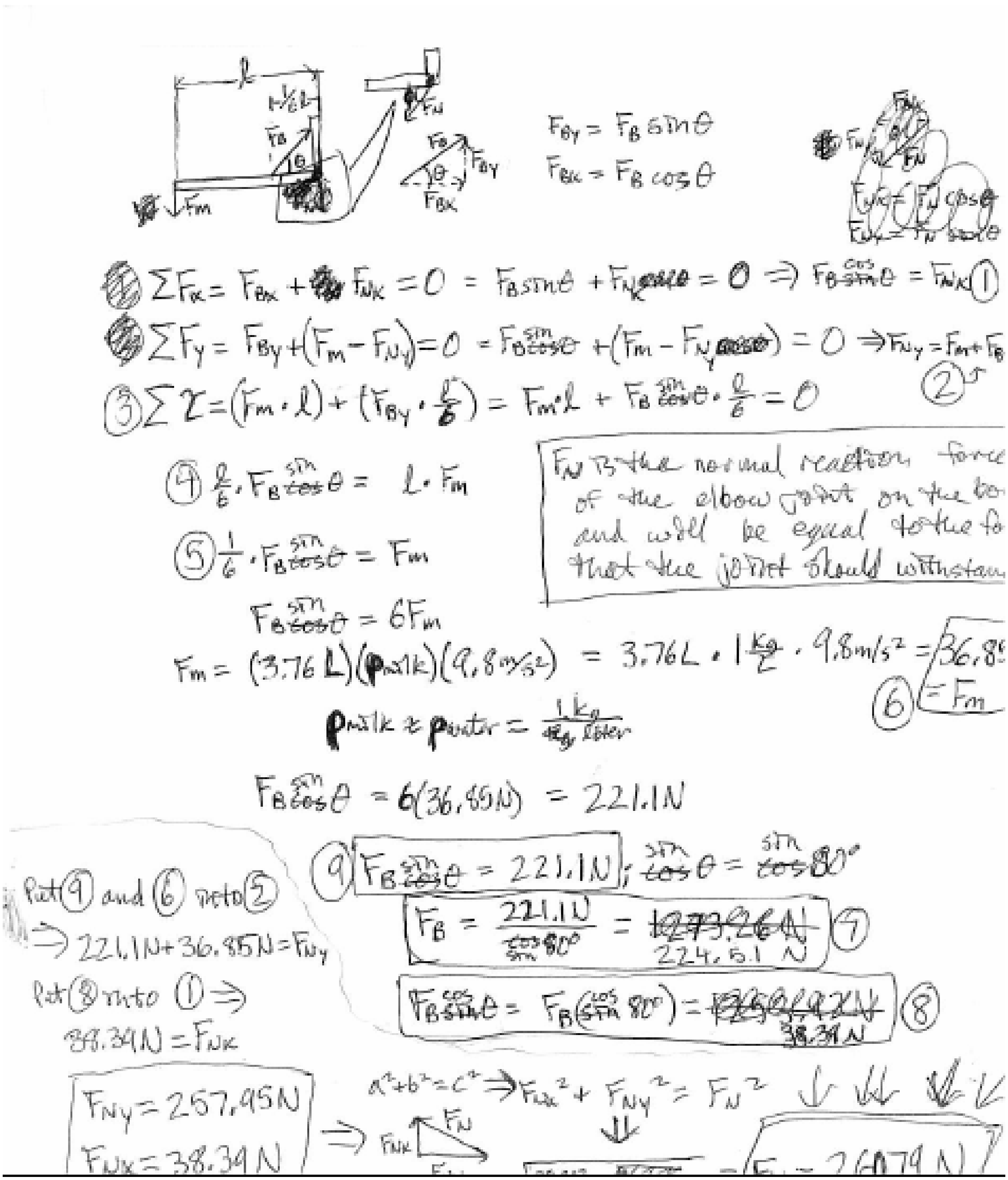
$F \times (\frac{1}{6}\ell) \times \cos 80° \geq - (3.76\,liter) g \ell$

$F \geq \dfrac{-(3.76\,liter)(9.81\,m/s^2)\ell}{\frac{1}{6}\ell \cos 80°}$

$F \geq 212\,N$

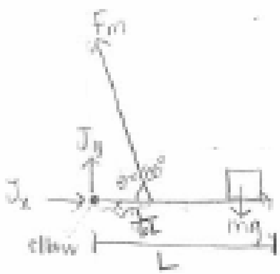
The joint must be able to handle at least this much force.

STUDENT #4:


$F_{By} = F_B \sin\theta$
$F_{Bx} = F_B \cos\theta$
$\sum F_x = F_{Bx} + F_{Nx} = 0 = F_B \sin\theta + F_{Nx} = 0 \Rightarrow F_B \frac{\cos}{\sin}\theta = F_{Nx}$ (1)
$\sum F_y = F_{By} + (F_m - F_{Ny}) = 0 = F_B \frac{\sin}{\cos}\theta + (F_m - F_{Ny}) = 0 \Rightarrow F_{Ny} = F_m + F_B$ (2)
(3) $\sum \tau = (F_m \cdot l) + (F_{By} \cdot \frac{l}{6}) = F_m l + F_B \frac{\sin}{\cos}\theta \cdot \frac{l}{6} = 0$
(4) $\frac{l}{6} \cdot F_B \frac{\sin}{\cos}\theta = l \cdot F_m$
(5) $\frac{1}{6} \cdot F_B \frac{\sin}{\cos}\theta = F_m$
F_N is the normal reaction force of the elbow joint on the bo[ne] and will be equal to the fo[rce] that the joint should withstan[d]
$F_B \frac{\sin}{\cos}\theta = 6F_m$
$F_m = (3.76\,L)(\rho_{milk})(9.8\,m/s^2) = 3.76L \cdot 1\frac{kg}{L} \cdot 9.8\,m/s^2 = 36.85$ (6) $= F_m$
$\rho_{milk} \approx \rho_{water} = \frac{1\,kg}{liter}$
$F_B \frac{\sin}{\cos}\theta = 6(36.85N) = 221.1N$
(9) $F_B \frac{\sin}{\cos}\theta = 221.1N$; $\frac{\sin}{\cos}\theta = \frac{\sin}{\cos}80°$
$F_B = \frac{221.1N}{\frac{\cos}{\sin}80°} = 224.51\,N$ (7)
$F_B \frac{\cos}{\sin}\theta = F_B(\frac{\cos}{\sin}80°) = 38.34\,N$ (8)
Put (9) and (6) into (2)
$\Rightarrow 221.1N + 36.85N = F_{Ny}$
Put (8) into (1) $\Rightarrow$
$38.34N = F_{Nx}$
$F_{Ny} = 257.95N$
$F_{Nx} = 38.34\,N$
$a^2 + b^2 = c^2 \Rightarrow F_{Nx}^2 + F_{Ny}^2 = F_N^2$
$F_N = 260.79\,N$

STUDENT #5:

The objective of the problem is to determine the force of the elbow joint so that it can support 3.76L while lower arm is in horizontal.



$\cos\theta = \frac{x}{F_m}$

$\sin\theta = \frac{J_x}{F_m}$

$T = F \cdot d\sin\theta$

$3.76 \text{ liter} \left(\frac{10^{-3} m^3}{1 \text{ liter}}\right)\left(\frac{10^3 kg}{m^3}\right) = 3.76 kg$

$T_{joint} = 0$

$T_{muscle} = F_m \cdot \frac{1}{6}L \sin 80$

$F_{elbow} = mg$

$F_{elbow} = 3.76 kg \cdot 9.8 \frac{m}{s^2} \approx 37 N$

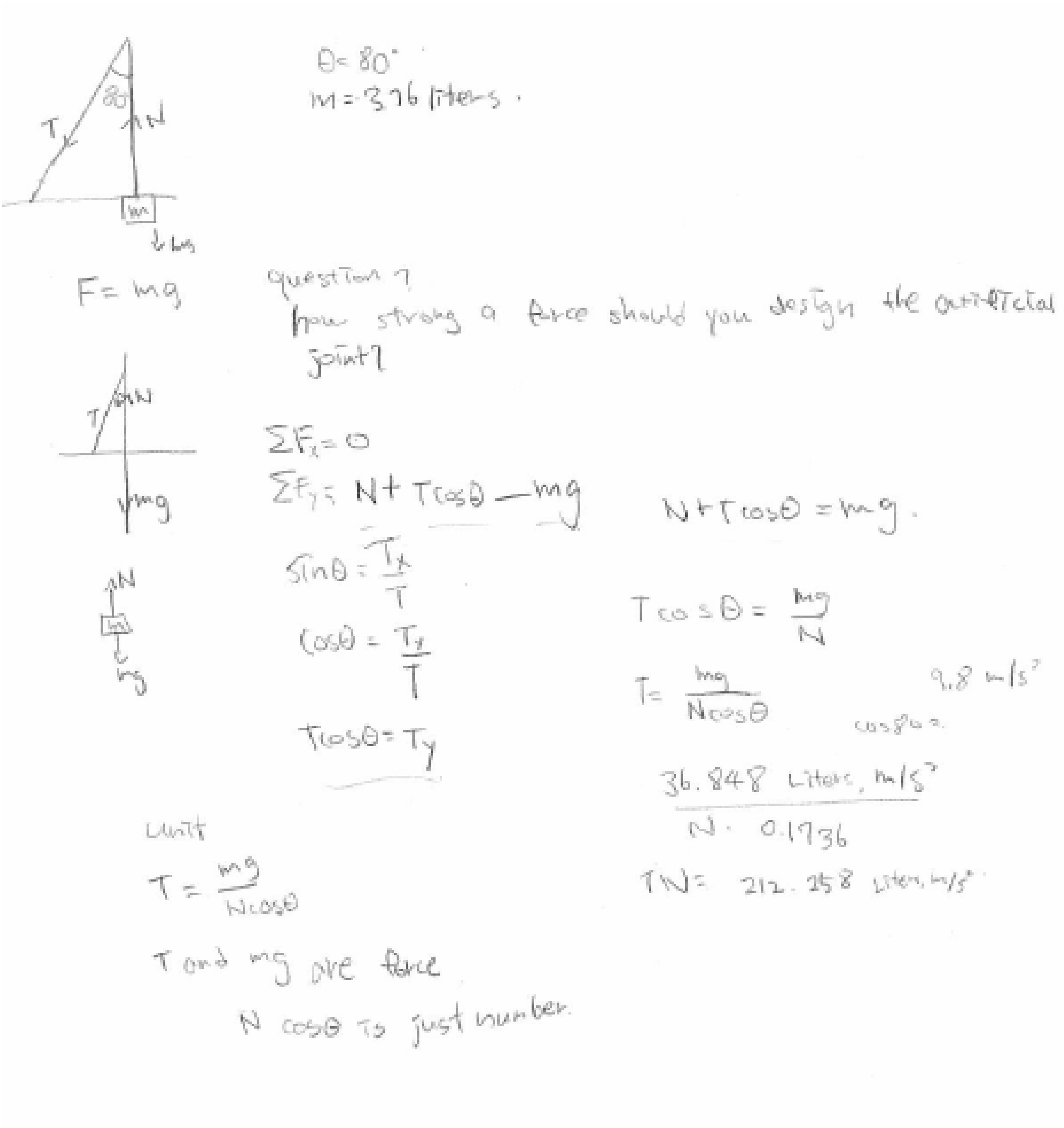
The elbow show be able to withstand 37N.

That seems reasonable. $37N \approx 8$ lbs.

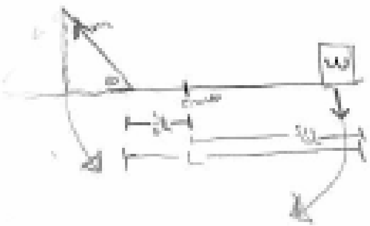
Unit Analysis

$F = kg \cdot \frac{m}{s^2} = \frac{kg \cdot m}{s^2} = N$

STUDENT #6:


θ= 80°
m= 376 liters.
T
80
N
m
F= mg
question 7
how strong a force should you design the anti-elastical joint?
T
N
mg
ΣFx = 0
ΣFy = N+ Tcosθ — mg
N+ Tcosθ = mg.
sinθ = Tx/T
cosθ = Ty/T
Tcosθ = Ty
N
m
mg
Tcosθ = mg/N
T= mg/Ncosθ
9.8 m/s²
cos80 =
36.848 Liters, m/s²
N · 0.1736
TN= 212.258 liters, m/s²
Unit
T = mg/Ncosθ
T and mg are force
N cosθ is just number.

STUDENT #7:



Known

W [illegible]

$\theta = 80°$

1 gallon approximately 8 lbs

$\tau = rF\sin\theta$

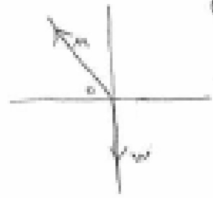
Approach: Find M as a function of the angle it attaches to the [illegible] [illegible] the patient must hold.

Question: How big should M be?

Target Quantity: M

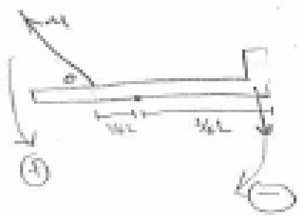
F_x	$F_y = 0$
$M\cos\theta$	$M\sin\theta$
	$-W$

$M\sin\theta - W = 0$



$\tau = rF\sin\theta$

torque balance



$-\left(\frac{5}{6}L\right)W$ = torque of milk

$+\left[\frac{1}{6}L\right]M\sin\theta$ = torque of [illegible] muscle

$$\left(\frac{1}{6}L\right)M\sin\theta - \left(\frac{5}{6}L\right)W = 0$$

$$M = \frac{\left(\frac{5}{6}L\right)W}{\left(\frac{1}{6}L\right)\sin\theta} = 40.6 \text{ lbs}$$

Unit check

$$\frac{(m)(lb)}{(m)} = lbs$$

(so it works)

$\sin\theta$ is unitless

- The bigger the mass of W and the farther away it is from the pivot point means you need a stronger bicep.

STUDENT #8:

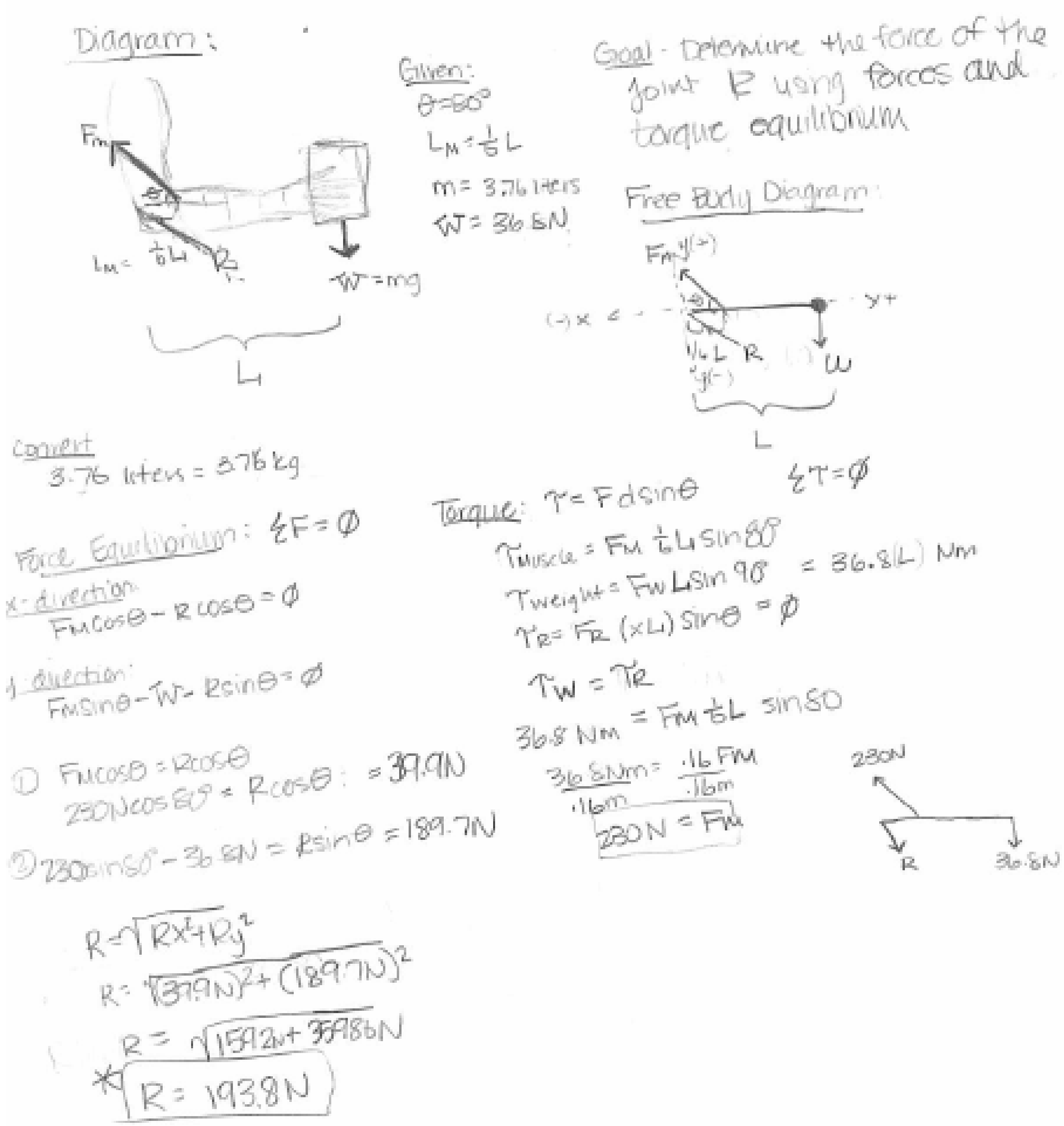


Diagram:
Given:
θ=80°
$L_M = \frac{1}{6}L$
m= 3.76 liters
W= 36.8N
Goal - Determine the force of the joint R using forces and torque equilibrium
Free Body Diagram
W=mg
L
Convert
3.76 liters = 3.76 kg
Force Equilibrium: ΣF=Ø
x-direction
FmCosθ - RcosΘ = Ø
y direction
FmSinθ - W - RsinΘ = Ø
① FmcosΘ = RcosΘ
230Ncos80° = RcosΘ = 39.9N
② 230sin80° - 36.8N = Rsinθ = 189.7N
Torque: τ= Fdsinθ ΣT=Ø
τmuscle = Fm ⅙L sin80°
τweight = Fw L sin 90° = 36.8(L) Nm
τR = FR (xL) sinθ = Ø
τW = τR
36.8 Nm = Fm ⅙L sin80
230N = Fm
230N
R
36.8N
R = √(Rx²+Ry²)
R = √((39.9N)²+(189.7N)²)
R = 1938N