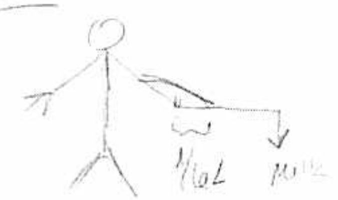
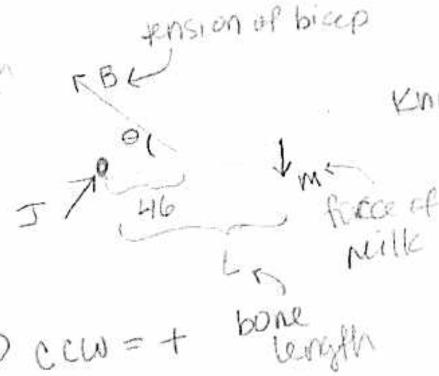
Student #1

Picture



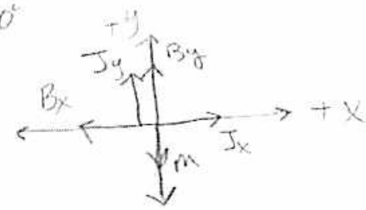
Diagram



↻ CCW = +

Know: M
$\theta = 80°$
L

Force Diagram



$Bx = B\cos\theta$ $\quad J = \sqrt{Jx^2 + Jy^2}$
$By = B\sin\theta$
$\tan\theta = By/Bx$

Approach
use forces $\Sigma F = 0$
use torques! $\Sigma \tau = 0$
ignore bone weight
By Newtons 3rd Law the force of the joint on the bone is equal to the force of the bone on the joint.

$\Sigma F_x = Jx - Bx = 0$
$\Sigma F_y = Jy + By - M = 0$
$\Sigma \tau = By\frac{L}{6} - ML = 0 \quad$ or $B\sin\theta = \frac{L}{6} - ML$

Target: J

			unknown
Find J	$J = \sqrt{Jx^2 + Jy^2}$	①	Jy, Jx
Find Jy	$Jy + By - M = 0$	②	By
Find Jx	$Jx - Bx = 0$	③	Bx
Find By	$By = B\sin\theta$	④	
Find Bx	$Bx = B\cos\theta$	⑤	

5 unknowns 5 equations

Put ⑤ into ③ and solve for Jx
$Bx = B\cos\theta, \quad Jx = B\cos\theta$

put ④ into ② and solve for Jy
$By = \sin\theta, \quad Jy = -\sin\theta + M$

now put Jy and Jx equations into ①
$J = \sqrt{(B\cos\theta)^2 + (-\sin\theta + M)^2}$

note that $B = \dfrac{\frac{L}{6} - ML}{\sin\theta}$
(from the equation for $\Sigma\tau$)
putting this into the equation for J:

$$J = \sqrt{\left(\left(\frac{\frac{L}{6} - ML}{\sin\theta}\right)\cos\theta\right)^2 + (-\sin\theta + M)^2}$$

We know that $\theta = 80°$ and the M is a gallon of milk but not its weight or mass (gallon is a unit of volum
so putting in θ gives:

$$J = \sqrt{\left(\left(\frac{\frac{L}{6} - ML}{\sin(80)}\right)\cos(80)\right)^2 + (-\sin 80 + M$$

check units:
$J = \sqrt{([\text{length}] - [\text{length}][\text{force}])^2 + [\text{force}]^2}$
$J = \sqrt{(\text{force})^2 + (\text{force})^2}$
J = force OK units check

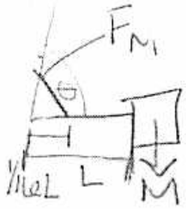
Student #2

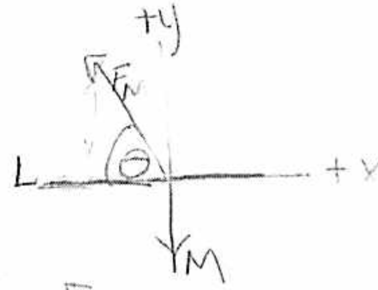
KNOW:

M - 3.76 LITERS

θ - 80°

$L_2 = 1/16\ L_1$





WEIGHT OF BONE NEGLIGIBLE.

Fm = FORCE OF MUSCLE

APPROACH:

Q. HOW STRONG A FORCE SHOULD THE ARTIFICLE JOINT BE MADE

USE FORCES NEGLECT BONE MASS

$\sum_x = L_x \cos\theta - M_x = 0$ $\quad \sum = F_{m_y} \sin\theta - M_y - 0$

$\tau = rF_\perp$ $\quad \sum\tau = F\cos\theta - m = 0$

FORCE OF MUSCLE $F_{M_x} = M_x - L_x$

FIND Fm

$F_m = M_x - L_x$

$L_x \cos\theta - M_x$

$M_x = L_x \cos\theta$

Fm

M_x

L_x

$L_x \cos\theta - F_{m_y} \sin\theta - F\cos\theta$

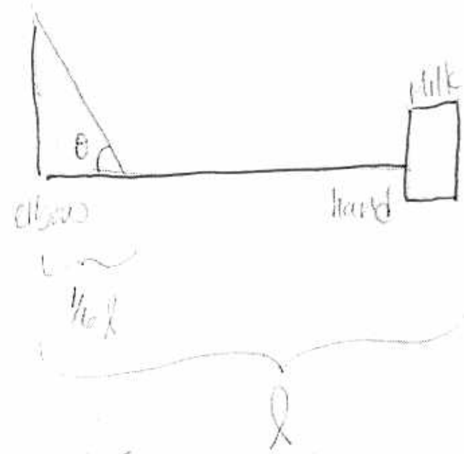
$L_x \cos\theta - F\cos\theta = F_{M_y} \sin\theta$

$$F_m = \frac{L_x \cos\theta - F\cos\theta}{\sin\theta}$$

Student #3

<u>Known</u>

- bicep muscle attached to bone at distance 1/6 bone length away from elbow joint
- $\theta = 80°$



milk = 3.76 liters
*don't know how to convert liters to grams...

$W = F \times \Delta x \times \cos\theta$

<u>Find</u>

Force

$$W_{muscle} \geq - W_{gravity}$$

$$F \times \Delta x \times \cos\theta \geq - mg \times \Delta x$$

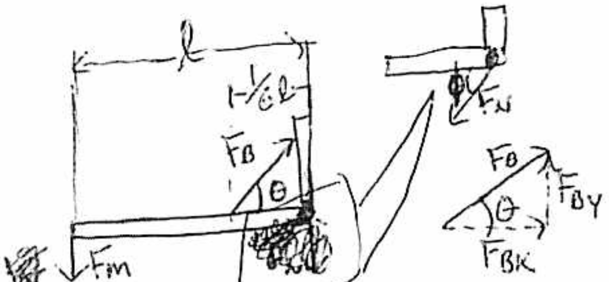
$$F \times (\tfrac{1}{6}\ell) \times \cos 80° \geq - (3.76 \text{ liter}) g \ell$$

$$F \geq \frac{-(3.76 \text{ liter})(9.81 \, m/s^2)\ell}{\tfrac{1}{6}\ell \cos 80°}$$

$$\boxed{F \geq 212 \text{ N}}$$

The joint must be able to handle at least this much force.

Student #4



$F_{By} = F_B \sin\theta$

$F_{Bx} = F_B \cos\theta$

$F_{Nx} = F_N \cos\theta$

$F_{Ny} = F_N \sin\theta$

(1) $\sum F_x = F_{Bx} + F_{Nx} = 0 = F_B \sin\theta + F_{Nx} = 0 \Rightarrow F_B \overset{\cos}{\cancel{\sin}}\theta = F_{Nx}$ (1)

(2) $\sum F_y = F_{By} + (F_m - F_{Ny}) = 0 = F_B \overset{\sin}{\cancel{\cos}}\theta + (F_m - F_{Ny}) = 0 \Rightarrow F_{Ny} = F_m + F_B$ (2)

(3) $\sum \tau = (F_m \cdot l) + (F_{By} \cdot \frac{l}{6}) = F_m l + F_B \overset{\sin}{\cancel{\cos}}\theta \cdot \frac{l}{6} = 0$

(4) $\frac{l}{6} \cdot F_B \overset{\sin}{\cancel{\cos}}\theta = l \cdot F_m$

(5) $\frac{1}{6} \cdot F_B \overset{\sin}{\cancel{\cos}}\theta = F_m$

$F_B \overset{\sin}{\cancel{\cos}}\theta = 6F_m$

F_N is the normal reaction force of the elbow joint on the bo
and will be equal to the fo
that the joint should withstan

$F_m = (3.76\,L)(\rho_{milk})(9.8\,m/s^2) = 3.76L \cdot 1\frac{kg}{L} \cdot 9.8\,m/s^2 = \boxed{36.8}$

(6) $\boxed{= F_m}$

$\rho_{milk} \approx \rho_{water} = \frac{1\,kg}{liter}$

$F_B \overset{\sin}{\cancel{\cos}}\theta = 6(36.85N) = 221.1N$

(9) $\boxed{F_B \overset{\sin}{\cancel{\cos}}\theta = 221.1N}$; $\overset{\sin}{\cancel{\cos}}\theta = \overset{\sin}{\cancel{\cos}}80°$

Put (9) and (6) into (2)

$\Rightarrow 221.1N + 36.85N = F_{Ny}$

$\boxed{F_B = \frac{221.1N}{\underset{\sin}{\cancel{\cos}}80°} = \cancel{1273.26N}\ 224.51\,N}$ (7)

Put (8) into (1) $\Rightarrow$

$38.39N = F_{Nx}$

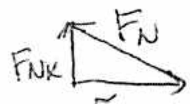
$\boxed{F_B \overset{\cos}{\cancel{\sin}}\theta = F_B(\overset{\cos}{\cancel{\sin}}\,80°) = 38.39\,N}$ (8)

$a^2+b^2=c^2 \Rightarrow F_{Nx}^2 + F_{Ny}^2 = F_N^2$

$F_{Ny} = 257.95N$

$F_{Nx} = 38.39\,N$

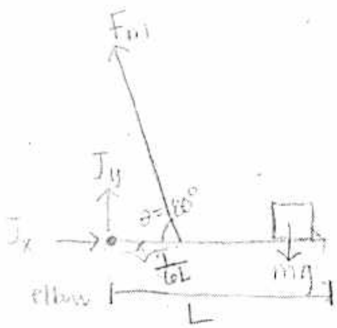
$\Rightarrow$



$F_N = 260.79\,N$

Student #5

The objective of the problem is to determine the force of the elbow joint so that it can support 3.76L while lower arm is in horizontal



$\cos\theta = \frac{X}{F_m}$

$\sin\theta = \frac{J_y}{F_m}$

$T = F \cdot d \sin\theta$

$3.76 \text{ liter} \left(\frac{10^{-3} m^3}{1 \text{ liter}}\right)\left(\frac{10^3 kg}{m^3}\right) = 3.76 kg$

$T_{joint} = 0$

$T_{muscle} = F_m \cdot \frac{1}{6}L \sin 80$

$F_{elbow} = mg$

$F_{elbow} = 3.76 kg \cdot 9.8 \frac{m}{s^2} \approx 37 N$

The elbow shown be able to withstand 37N

That seems reasonable, $37 N \approx 8$ lbs.

Unit Analysis

$F = kg \cdot \frac{m}{s^2} = \frac{kg \cdot m}{s^2} = N$

Student # 6

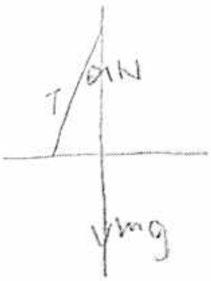
$\theta = 80°$

$m = 3.76$ liters.

$F = mg$

question ?

how strong a force should you design the artificial joint?



$\Sigma F_x = 0$

$\Sigma F_y = N + T\cos\theta - mg$

$N + T\cos\theta = mg$.

$\sin\theta = \frac{T_x}{T}$

$\cos\theta = \frac{T_y}{T}$

$T\cos\theta = T_y$

$T\cos\theta = \frac{mg}{N}$

$T = \frac{mg}{N\cos\theta}$

9.8 m/s²

cos80 =

$\frac{36.848 \text{ Liters. m/s}^2}{N \cdot 0.1736}$

TN = 212.258 Liters m/s²

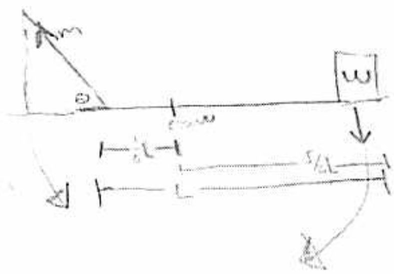
unit

$T = \frac{mg}{N\cos\theta}$

T and mg are force

N cosθ is just number

Student # 7



Known
W 3.76 Liters
$\theta = 80°$
1 gallon approximately 8 lbs

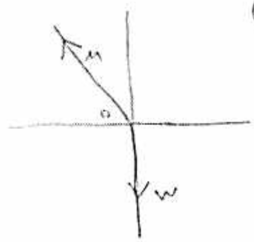
$\tau = rF\sin\theta$

Approach: find M as a function of the angle it attaches to the bone & the weight the patient must hold.

Question: How big should M be?

Fx:	Fy = 0
$M\cos\theta$	$M\sin\theta$
	$-W$

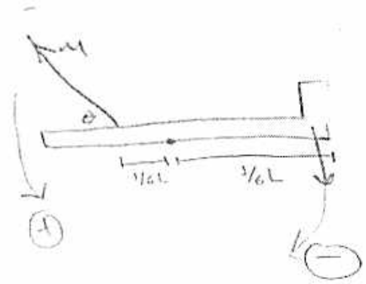
$M\sin\theta - W = 0$



Target Quantity: M

$\tau = rF\sin\theta$

torque balancing



$-\left(\frac{5}{6}L\right)W$ = torque of milk

$+\left[\frac{1}{6}L\right]M\sin\theta$ - torque of bonemuscle

$$\left(\frac{1}{6}L\right)M\sin\theta - \left(\frac{5}{6}L\right)W = 0$$

$$M = \frac{\left(\frac{5}{6}L\right)W}{\left(\frac{1}{6}L\right)\sin\theta} = 40.6\ \text{lbs}$$

Unit check

$$\frac{(m)(lb)}{(m)} = lbs$$

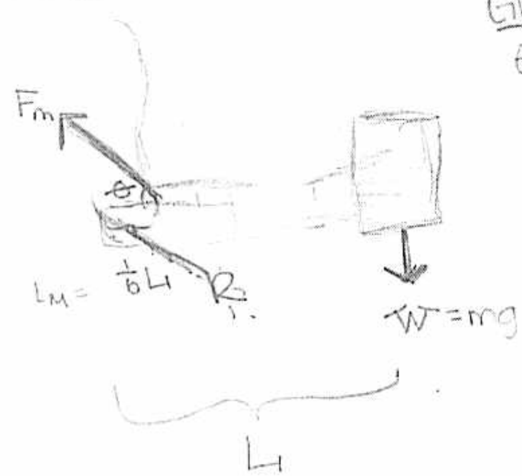
(so it works)

$\sin\theta$ is unitless

- the bigger the mass of W and the farther away it is from the pivot point means you need a stronger bicep.

Student # 8

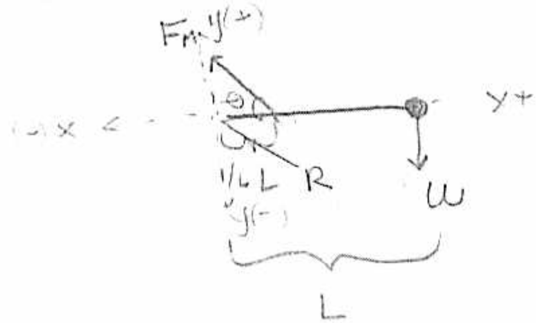
Diagram:



Given:
$\theta = 80°$
$L_M = \frac{1}{6}L$
m = 3.76 liters
W = 36.8N

Goal: Determine the force of the joint R using forces and torque equilibrium

Free Body Diagram



convert
3.76 liters = 3.76 kg

Force Equilibrium: $\Sigma F = \emptyset$

x-direction
$F_M \cos\theta - R\cos\theta = \emptyset$

y-direction
$F_M \sin\theta - W - R\sin\theta = \emptyset$

① $F_M \cos\theta = R\cos\theta$
$230N \cos 80° = R\cos\theta \quad = 39.9N$

② $230 \sin 80° - 36.8N = R\sin\theta = 189.7N$

$R = \sqrt{Rx^2 + Ry^2}$
$R = \sqrt{(37.9N)^2 + (189.7N)^2}$
$R = \sqrt{1592N + 35986N}$
$\boxed{R = 193.8N}$

Torque: $\tau = Fd\sin\theta$ $\qquad \Sigma\tau = \emptyset$

$\tau_{Muscle} = F_M \frac{1}{6}L_1 \sin 80$
$\tau_{Weight} = F_W L_1 \sin 90 \quad = 36.8L \ Nm$
$\tau_R = F_R (xL_1) \sin\theta = \emptyset$

$\tau_W = \tau_R$
$36.8\ Nm = F_M \frac{1}{6}L \sin 80$
$\frac{36.8Nm}{.16m} = \frac{.16 F_M}{.16m}$
$\boxed{230N = F_M}$