What Are The Characteristics of a Good Group Problem?

Group problems should be designed to encourage students to use an organized, logical problem-solving strategy instead of their novice, formula-driven, "plug-and-chug" strategy. Specifically, they should encourage students to (a) consider physics concepts in the context of real objects in the real world; (b) view problem-solving as a series of decisions; and (c) use their conceptual understanding of the fundamental concepts of physics to qualitatively analyze a problem *before* the mathematical manipulation of formulas.

Group problems should be more difficult to solve than easy problems typically given on an individual test. But the increased difficulty should be primarily conceptual, not mathematical. Difficult mathematics is best accomplished by individuals, not by groups. So problems that involve long, tedious mathematics but little physics, or problems that require the use of a shortcut or "trick" that only experts would be likely to know do not make good group problems. In fact, the best group problems involve the straight-forward application of the fundamental principles (e.g., the definition of velocity and acceleration, the independence of motion in the vertical and horizontal directions) rather than the repeated use of derived formulas (e.g., $v_f^2 - v_o^2 = 2ad$).

There are twenty-one characteristics of a problem that can make it more difficult to solve than a standard textbook exercise:

Approach

1 Cues Lacking

A. No explicit target variable. The unknown variable of the problem is not explicitly stated.

B. Unfamiliar context. The context of the problem is very unfamiliar to the students (e.g., cosmology, molecules).

2 Agility with Principles

A. Choice of useful principles. The problem has more than one possible set of useful concepts that could be applied for a correct solution.

B. Two general principles. The correct solution requires students to use two major principles (e.g., torque and linear kinematics).

C. Very abstract principles. The central concept in the problem is an abstraction of another abstract concept. (e.g., potential, magnetic flux).

3 Non-standard Application

A. Atypical situation. The setting, constraints, or complexity is unusual compared with textbook problems.

- Unusual target variable. The problem involves an atypical target variable when compared with homework problems.

Analysis of Problem

4 Excess or Missing Information

A. Excess numerical data. The problem statement includes more data than is needed to solve the problem.

B. Numbers must be supplied. The problem requires students to either remember or estimate a number for an unknown variable.

C. Simplifying assumptions. The problem requires students to generate a simplifying assumption to eliminate an unknown variable.

5 Seemingly Missing Information

A. Vague statement. The problem statement introduces a vague, new mathematical statement.

B. Special conditions or constraints. The problem requires students to generate information from their analysis of the conditions or constraints.

C. Diagrams. The problem requires students to extract information from a spatial diagram.

6 Additional Complexity

A. More than two subparts. The problem solution requires students decompose the problem into more than two subparts.

B. Five or more terms per equation. The problem involves five or more terms in a principle equation (e.g., three or more forces acting along one axes on a single object).

C. Two directions (vector components). The problem requires students to treat principles (e.g., forces, momentum) as vectors.

Mathematical Solution

7 Algebra Required

A. No numbers. The problem statement does not use any numbers.

B. Unknown(s) cancel. Problems in which an unknown variable, such as a mass, ultimately factors out of the final solution.

C. Simultaneous equations. A problem that requires simultaneous equations for a solution.

8 Targets Math Difficulties

A. Calculus or vector algebra. The solution requires the students to sophisticated vector algebra, such as cross products, or calculus.

B. Lengthy or Detailed Algebra. A successful solution to the problem is not possible without working through lengthy or detailed algebra (e.g., a messy quadratic equation).

BEWARE! Good group problems are difficult to construct because they can easily be made too complex and difficult to solve. A good group problem does not have **all** of the above difficulty characteristics, but usually only *2- 5* of these characteristics.

How to Create Context-rich Group Problems

One way to invent group problems is to start with a textbook exercise or problem, then modify the problem. You may find the following steps helpful:

1. If necessary, determine a context (real objects with real motions or interactions) for the textbook exercise or problem. You may want to use an unfamiliar context for a **very** difficult group problem.

2. Decide on a motivation -- Why would anyone want to calculate something in this context?

3. Determine if you need to change the target variable to

   (a) make the problem more than a one-step exercise, or

   (b) make the target variable fit your motivation.

5. Determine if you need to change the given information (or target variable) to make the problem an application of fundamental principles (e.g., the definition of velocity or acceleration) rather than a problem needing the application of many derived formulas.

4. Write the problem like a short story.

5. Decide how many "difficulty" characteristics (characteristics that make the problem more difficult) you want to include, then do some of the following:
   (a) think of an unfamiliar context; or use an atypical setting or target variable;
   (b) think of different information that could be given, so two approaches (e.g., kinematics *and* forces) would be needed to solve the problem instead of one approach (e.g., forces), or so that more than one approach could be taken
   (c) write the problem so the target variable is not explicitly stated;
   (d) determine extra information that someone in the situation would be likely to have; or leave out common-knowledge information (e.g., the boiling temperature of water);
   (e) depending on the context, leave out the explicit statement of some of the problem idealizations (e.g., change "massless rope" to "very light rope"); or remove some information that students could extract from an analysis of the situation;
   (f) take the numbers out of the problem and use variable names only;
   (g) think of different information that could be given, so the problem solution requires the use of vector components, geometry / trigonometry to eliminate an unknown, or calculus.

6. Check the problem to make sure it is solvable, the physics is straight-forward, and the mathematics is reasonable.

Some common contexts include:

- physical work (pushing, pulling, lifting objects vertically, horizontally, or up ramps)
- suspending objects, falling objects
- sports situations (falling, jumping, running, throwing, etc. while diving, bowling, playing golf, tennis, football, baseball, etc.)
- situations involving the motion of bicycles, cars, boats, trucks, planes, etc.
- astronomical situations (motion of satellites, planets)
- heating and cooling of objects (cooking, freezing, burning, etc.)

Sometimes it is difficult to think of a motivation. We have used the following motivations:

- You are . . . . (in some everyday situation) and need to figure out . . . .
- You are watching . . . . (an everyday situation) and wonder . . . .
- You are on vacation and observe / notice . . . . and wonder . . . .
- You are watching TV or reading an article about . . . . and wonder . . .
- Because of your knowledge of physics, your friend asks you to help him / her . . . .
- You are writing a science-fiction or adventure story for your English class about . . . . and need to figure out . . . .
- Because of your interest in the environment and your knowledge of physics, you are a member of a Citizen's Committee (or Concern Group) investigating . . . .
- You have a summer job with a company that . . . . Because of your knowledge of physics, your boss asks you to . . . .
- You have been hired by a College research group that is investigating . . . . Your job is to determine . . . .
- You have been hired as a technical advisor for a TV (or movie) production to make sure the science is correct. In the script . . . ., but is this correct?
- When really desperate, you can use the motivation of an artist friend designing a kinetic sculpture!

Decision Strategy for Judging Problems

Outlined below is a decision strategy to help you decide whether a context-rich problem is a good individual test problem, group practice problem, or group test problem.

1. *Read* the problem statement. *Draw* the diagrams and *determine* the equations needed to solve the problem (through plan-a-solution step).

2. *Reject* if:
   - the problem can be solved in one step,
   - the problem involves long, tedious mathematics, but little physics; or
   - the problem can only be solved easily using a "trick" or shortcut that only experts would be likely to know. (In other words, the problem should be a straight-forward application of fundamental concepts and principles.)

3. *Check* for the twenty-one characteristics that make a problem more difficult:

Approach	Analysis	Mathematical Solution
• Cues Lacking ___ A. No target variable ___ B. Unfamiliar context • Agility with Principles ___ A. Choice of principle ___ B. Two principles ___ C. Abstract principle • Non-Standard Application ___ A. Atypical situation ___ B. Unusual target	• Excess or Missing Info. ___ A. Excess data ___ B. Numbers required ___ C. Assumptions • Seemingly Missing Info. ___ A. Vague statement ___ B. Special constraints ___ C. Diagrams • Additional Complexity ___ A. >2 subparts ___ B. 5+ terms ___ C. Vectors	• Algebra required ___ A. No numbers ___ B. Unknown(s) cancel ___ C. Simultaneous eqns. • Targets Math Difficulty ___ A. Calc/vector algebra ___ B. Lengthy algebra

4. *Decide* if the problem would be a good group practice problem (20 - 25 minutes), a good group test problem (45 - 50 minutes), or a good (easy, medium, difficult) individual test problem, depending on three factors: (a) the complexity of mathematics, (b) the timing (when problem is to be given to students), and (c) the number of difficulty characteristics of the problem:

Type of Problem	Timing	Diff. Ch.
Group Practice Problems should be shorter and mathematically easier than group test problems.	just introduced to concept(s) just finished study of concept(s)	2 - 3 3 - 4
Group Test Problems can be more complex mathematically.	just introduced to concept(s) just finished study of concept(s)	3 - 4 4 - 5

Type of Problem	Timing	Diff. Ch.
Individual Problems can be easy, medium-difficult, or difficult:		
Easy	just introduced to concept(s) just finished study of concept(s)	0 -1 1 - 2
Medium-difficult	just introduced to concept(s) just finished study of concept(s)	1 - 2 2 - 3
Difficult	just introduced to concept(s) just finished study of concept(s)	2 - 3 3 - 4

There is considerable overlap in the criteria, so most problems can be judged to be **both** a good group practice or test problem *and* a good easy, medium-difficult, or difficult individual problem.

Examples of how to judge context-rich problems

Example Problem #1:
You are helping your friend prepare for her next skate board exhibition. For her program, she plans to take a running start and then jump onto her heavy duty 15-lb stationary skateboard. She and the skateboard will glide in a straight line along a short, level section of track, then up a sloped concrete wall. She wants to reach a height of at least 10 feet above where she started before she turns to come back down the slope. She has measured her maximum running speed to safely jump on the skateboard at 7 feet/second. She knows you have taken physics, so she wants you to determine if she can carry out her program as planned. She tells you that she weighs 100 lbs.

Assume that students have just started to study the conservation of energy and conservation of momentum.

The approach to solve this problem involves using conservation of energy and conservation of momentum.

Should we reject it?

- The problem can be solved in one step. NO.
- The problem involves long, tedious mathematics, but little physics. NO.
- The problem can only be solved easily using a "trick" or shortcut that only experts would be likely to know. (In other words, the problem should be a straight-forward application of fundamental concepts and principles.) NO.

Which of the characteristics does this problem involve?

Approach	Analysis	Mathematical Solution
1. Cues Lacking	4. Excess or Missing Info.	7. Algebra required
_v_ A. No target variable	___ A. Excess data	___ A. No numbers
___ B. Unfamiliar context	___ B. Numbers required	___ B. Unknown(s) cancel
	_v_ C. Assumptions	___ C. Simultaneous eqns.
2. Agility with Principles		
___ A. Choice of principle	5. Seemingly Missing Info.	8. Targets Math Difficulty
_v_ B. Two principles	___ A. Vague statement	___ A. Calc/vector algebra
___ C. Abstract principle	___ B. Special constraints	___ B. Lengthy algebra
	___ C. Diagrams	
3. Non-Standard Application		
___ A. Atypical situation	6. Additional Complexity	
___ B. Unusual target	___ A. >2 subparts	
	___ B. 5+ terms	
	___ C. Vectors	

This has a difficulty rating of 3, two of which are in the approach. The mathematics involved is easy. This would make a decent group practice problem or a medium-difficult individual test problem. It is too easy for a group test problem.

If teaching this as a group practice problem, you could expect students to spend more time on the setup of the problem, and less time on the math.

Example Problem #2:

Electric and Gravitational Force: You and a friend are reading a newspaper article about nuclear fusion energy generation in stars. The article describes the helium nucleus, made up of two protons and two neutrons, as very stable so it doesn't decay. You immediately realize that you don't understand why the helium nucleus is stable. You know that the proton has the same charge as the electron except that the proton charge is positive. Neutrons you know are neutral. Why, you ask your friend, don't the protons simply repel each other causing the helium nucleus to fly apart? Your friend says she knows why the helium nucleus does not just fly apart. The gravitational force keeps it together, she says. Her model is that the two neutrons sit in the center of the nucleus and gravitationally attract the two protons. Since the protons have the same charge, they are always as far apart as possible on opposite sides of the neutrons. What mass would the neutron have if this model of the helium nucleus works? Is that a reasonable mass? Looking in your physics book, you find that the mass of a neutron is about the same as the mass of a proton and that the diameter of a helium nucleus is 3.0×10^{-13} cm.

Assume that students have just finished studying electric forces.

The approach to solve this problem involves using the idea of electric force and gravitational force.

Should we reject it?

- The problem can be solved in one step. NO.
- The problem involves long, tedious mathematics, but little physics. NO.

- The problem can only be solved easily using a "trick" or shortcut that only experts would be likely to know. (In other words, the problem should be a straight-forward application of fundamental concepts and principles.) NO.

Which of the characteristics does this problem involve?

Approach	Analysis	Mathematical Solution
1. Cues Lacking _v_ A. No target variable _v_ B. Unfamiliar context 2. Agility with Principles ___ A. Choice of principle _v_ B. Two principles ___ C. Abstract principle 3. Non-Standard Application _v_ A. Atypical situation ___ B. Unusual target	4. Excess or Missing Info. ___ A. Excess data ___ B. Numbers required ___ C. Assumptions 5. Seemingly Missing Info. ___ A. Vague statement ___ B. Special constraints _v_ C. Diagrams 6. Additional Complexity ___ A. >2 subparts ___ B. 5+ terms ___ C. Vectors	7. Algebra required ___ A. No numbers ___ B. Unknown(s) cancel ___ C. Simultaneous eqns. 8. Targets Math Difficulty ___ A. Calc/vector algebra ___ B. Lengthy algebra

This has a difficulty rating of 5, four of which are in the approach. The mathematics involved is easy, but the difficulty is all in the setup. Students probably have not studied gravitational force lately, which makes the problem more difficult. This would be a difficult group test problem.
If teaching this as a group test problem, you could expect groups to spend most of their time on the setup of the problem, so don’t worry if they haven’t gotten to the math by the middle of the hour.