

Conservation of Energy

Conservation of Momentum

The problems in this section require conservation principles to solve -- the conservation of energy, the conservation of momentum, or both. For convenience, the problems are divided into four groups by the concepts required for a solution: (1) conservation of energy (mechanical, gravitational); (2) conservation of energy (mechanical) and force; (3) conservation of momentum; and (4) conservation of energy (mechanical) and conservation of momentum.

Conservation of Energy (Mechanical, Gravitational)

1. You are watching a National Geographic Special on television. One segment of the program is about archer fish, which inhabit streams in southeast Asia. This fish actually "shoots" water at insects to knock them into the water so it can eat them. The commentator states that the archer fish keeps its mouth at the surface of the stream and squirts a jet of water from its mouth at 13 feet/second. You watch an archer fish shoot a juicy moth off a leaf into the water. You estimate that the leaf was about 2.5 feet above a stream. You wonder at what minimum angle from the horizontal the water can be ejected from the fish's mouth to hit the moth. Since you have time during the commercial, you quickly calculate this angle.
2. Your artist friend is designing a kinetic sculpture and asks for your help since she knows that you have had physics. Part of her sculpture consists of a 6.0-kg object (you can't tell what it is supposed to be, but it's art) and a 4.0-kg object which hang straight down from opposite ends of a very thin, flexible wire. This wire passes over a smooth, cylindrical, horizontal, stainless steel pipe 3.0 meters above the floor. The frictional force between the rod and the wire is negligible. The 6.0-kg object is held 2.0 meters above the floor and the other object hangs 0.50 meters above the floor. When the mechanism releases the 6.0-kg object, both objects accelerate and one will eventually hit the floor -- but they don't hit each other. To determine if the floor will be damaged, calculate the speed of the object which hits the floor.
3. You are driving your car uphill along a straight road. Suddenly, you see a car run a red light and enter the intersection just ahead of you. You slam on your brakes and skid in a straight line to a stop, leaving skid marks 100 feet long. A policeman observes the whole incident and gives a ticket to the other car for running a red light. He also gives you a ticket for exceeding the speed limit of 30 mph. When you get home, you read your physics book and estimate that the coefficient of kinetic friction between your tires and the road was 0.60, and the coefficient of static friction was 0.80. You estimate that the hill made an angle of about 10° with the horizontal. You look in your owner's manual and find that your car weighs 2,050 lbs. Will you fight the traffic ticket in court?
4. You have landed a summer job with a company that has been given the contract to design the ski jump for the next Winter Olympics. The track is coated with snow and has an angle of 25° from the horizontal. A skier zips down the ski jump ramp so that he leaves it at high speed. The winner is the person who jumps the farthest after leaving the end of the ramp. Your task is to determine the height of the starting gate above the end of the ramp, which will determine the mechanical structure of the ski jump facility. You have been told that the typical ski-jumper pushes off from

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the starting gate at a speed of 2.0 m/s. For safety reasons, your design should be such that for a perfect run down the ramp, the skier's speed before leaving the end of the ramp and sailing through the air should be no more than 80 km/hr. You run some experiments on various skies used by the jumpers and determine that the coefficient of static friction between the snow and the skis is 0.10 and its coefficient of kinetic friction is 0.02. Since the ski-jumpers bend over and wear very aerodynamic suits, you decide to neglect the air resistance to make your design.

5. The Navy wants a new airplane launcher for their aircraft carriers and you are on the design team. The launcher is effectively a large spring that pushes the plane for the first 5 meters of the 20 meter long runway. During that same time, the plane's jet engines supply a constant thrust of 5.4×10^4 N for the entire length of the runway. The 2000 kg planes need to have a velocity of 45 m/s by the end of the runway. What should be the spring constant for the launcher?
6. You have been hired to design a safety system to protect drivers going down hills during an ice storm. The planned system consists of a bumper, which can be considered a stiff spring, at the bottom of the hill. In the scenario you are given, the car starts from rest at the top of a hill which makes an angle θ with the horizontal. The distance that the car slides from the top of the hill until it is stopped by the spring is L . For the worst case scenario, assume that there is no frictional force between the car and road due to the ice. If the maximum compression of the spring from its equilibrium position is D , your job is to calculate the required spring constant k in terms of D , L and θ .
7. You are the technical advisor to the Dave Letterman Show. Your task is to design a circus stunt in which Super Dave Osbourne, who weighs 170 pounds, is shot out of a cannon that is elevated 40° from the horizontal. The "cannon" is actually a 3-foot diameter tube that uses a stiff spring and a puff of smoke rather than an explosive to launch Super Dave. The manual for the cannon states that the spring constant is 1822 Newtons/meter. The spring is compressed by a motor until its free end is level with the bottom of the cannon tube, which is 5 feet above the ground. A small seat is attached to the free end of the spring for Super Dave to sit on. When the spring is released, it extends 9 feet up the tube. Neither the seat nor the chair touch the sides of the 12-foot long tube. After a drum roll, the spring is released and Super Dave will fly through the air with the appropriate sound effects and smoke. You have an airbag 3-feet thick for Super Dave to land on. You know that the airbag will exert an average retarding force of 2850 Newtons in all directions. You need to determine if the airbag is thick enough to stop Super Dave safely. -- that is, so he is slowed to a stop by the time he reaches ground level.
8. Super Dave has just returned from the hospital where he spent a week convalescing from injuries incurred when he was "shot" out of a cannon to land in an airbag which was too thin. Undaunted, he decides to celebrate his return with a new stunt. He intends to jump off a 100-foot tall tower with an elastic cord tied to one ankle, and the other end tied to the top of the tower. This cord is very light but very strong and stretches so that it can stop him without pulling his leg off. Such a cord exerts a force with the same mathematical form as the spring force. He wants it to be 75 feet long so that he will be in free fall for 75 feet before the cord begins to stretch. To minimize the force that the cord exerts on his leg, he wants it to stretch as far as possible. You have been assigned to purchase the cord for the stunt and must determine the elastic force constant which

characterizes the cord that you should order. Before the calculation, you carefully measure Dave's height to be 6.0 ft and his weight to be 170 lbs. For maximum dramatic effect, his jump will be off a diving board at the top of the tower. From tests you have made, you determine that his maximum speed coming off the diving board is 10 ft/sec. Neglect air resistance in your calculation -- let Dave worry about that.

9. You were so impressed with the problem about Super Dave that you decide that this would make a good stunt for the Institute of Technology (IT) day. To raise money for a University scholarship fund, you want to have the new IT dean bungee jump from a crane if contributions can be found for 10 scholarships. To add some interest, the jump will be made from 44 m above a 2.5 m deep pool of Jell-O. A 30-m long bungee cord would be attached to the dean's ankle. First you must convince the dean that your plan is safe for a person of his mass, 70 kg. The dean knows that as the bungee cord begins to stretch, it will exert a force which has the same properties as the force exerted by a spring. Your plan has the dean stepping off a platform and being in free fall for the 30 m before the cord begins to stretch. You must determine the elastic constant of the bungee cord so that it stretches only 12 m, which will just keep the dean's head out of the Jell-O.
10. Your friend is an artist. His new work is a kinetic sculpture called "destruction." The sculpture is simple and has high impact. A 200-kg steel block is hung from the ceiling at the end of an 8-foot long rope. Another rope is attached to the block so that it pulls it horizontally. The other end of the horizontal rope is attached to a motor which is cleverly mounted so that the rope always pulls the block horizontally with a constant force. The block starts from rest when it is hanging straight down and moves very slowly until it is hanging at an angle of 30° to the vertical. At that point the horizontal rope will be released and the block swings until it crashes into a wall. Your friend knows you have taken physics and asks you the minimum energy that the motor must supply. You make a test and determine that the block is in equilibrium when it is pulled by the horizontal rope connected to the motor and the block is hanging from the other rope at 30° from the vertical.
11. (**Gravitational Energy**) Because of your knowledge of physics and interest in the environment, you have gotten a summer job with an organization which wants to orbit a satellite to monitor the amount of chlorine ions in the upper atmosphere over North America. It has been determined that the satellite should collect samples at a height of 100 miles above the Earth's surface. Unfortunately, at that height air resistance would make the amount of time the satellite would stay in orbit too short to be useful. You suggest that an elliptical orbit would allow the satellite to be close to the Earth over North America, where data was desired, but farther from the Earth, and thus out of almost all of the atmosphere, on the other side of our planet. Your colleague estimates that the satellite would be traveling at 10,000 miles/hour when it was farthest from the Earth at a height of 1,000 miles. How fast would the satellite be traveling when it took its air samples if you neglect air friction?

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12. At the train station, you notice a large horizontal spring at the end of the track where the train comes in. This is a safety device to stop the train so that it will not go plowing through the station if the engineer misjudges the stopping distance. While waiting, you wonder what would be the fastest train that the spring could stop by being fully compressed, 3.0 ft. To keep the passengers as safe as possible when the spring stops the train, you assume that the maximum stopping acceleration of the train, caused by the spring, is $g/2$. You make a guess that a train might have a mass of 0.5 million kilograms. For the purpose of getting your answer, you assume that all frictional forces are negligible.
13. You have a summer job at a company that specializes in the design of equipment for sports shows and exhibitions. The company has been given the contract to design a piece of apparatus for an ice skating show. An ice skater will start from rest and glide down an ice-covered ramp. At the bottom of the ramp, the skater will continue gliding around in a ice-covered loop which is inside of a vertical circle. After going around the vertical circle, the skater emerges at the bottom of the circle to glide out on the skating rink floor to the wild applause of the audience. To make a spectacular effect, the circular loop should have a diameter of 30 feet. Your task is to determine the minimum height of the top of the ramp to the rink floor so that the skater will not fall off the loop at the top.
14. In a weak moment you have volunteered to be a human cannonball at an amateur charity circus. The "cannon" is actually a 3-foot diameter tube with a big stiff spring inside which is attached to the bottom of the tube. A small seat is attached to the free end of the spring. The ringmaster, one of your soon to be ex-friends, gives you your instructions. He tells you that just before you enter the mouth of the cannon, a motor will compress the spring to $1/10$ its normal length and hold it in that position. You are to gracefully crawl in the tube and sit calmly in the seat without holding on to anything. The cannon will then be raised to an angle such that your speed through the air at your highest point is 10 ft/sec. When the spring is released, neither the spring nor the chair will touch the sides of the 12-foot long tube. After the drum roll, the spring is released and you will fly through the air with the appropriate sound effects and smoke. With the perfect aim of your gun crew, you will fly through the air over a 15-foot wall and land safely in the net. You are just a bit worried and decide to calculate how high above your starting position you will be at your highest point. Before the rehearsal, the cannon is taken apart for maintenance. You see the spring, which is now removed from the cannon, is hanging straight down with one end attached to the ceiling. You determine that it is 10 feet long. When you hang on its free end without touching the ground, it stretches by 2.0 ft. Is it possible for you to make it over the wall?

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15. As a concerned citizen, you have volunteered to serve on a committee investigating injuries to Junior High School students participating in sports programs. Currently your committee is investigating the high incidence of ankle injuries on the basketball team. You are watching the team practice, looking for activities which can result in large horizontal forces on the ankle. Observing the team practice jump shots gives you an idea, so you try a small calculation. A 40-kg student

jumps 1.0 meters straight up and shoots the 0.80-kg basketball at his highest point. From the trajectory of the basketball, you deduce that the ball left his hand at 30° from the horizontal at 20 m/s. What is his horizontal velocity when he hits the ground?

16. You are a volunteer at the Campus Museum of Natural History. Because of your interest in the environment and your physics experience, you have been asked to assist in the production of an animated film about the survival of hawks in the wilderness. In the script, a 1.5-kg hawk is hovering in the air so it is stationary with respect to the ground when it sees a goose flying below it. The hawk dives straight down. When it strikes the goose and digs its claws into the goose's body, it has a speed of 60 km/hr. The goose, which has a mass of 2.5 kg, was flying north at 30 km/hr just before it was struck by the hawk and killed instantly. The animators want to know the velocity (magnitude and direction) of the hawk and dead goose just after the strike.
17. You are looking forward to the end of final exams with more anticipation than usual because you have lined up a great summer job. You might be hired by a company searching for treasure in the Caribbean! Your prospective employer has discovered the captain's logbook of the 40,000-ton luxury liner, the Hedonist, which left Miami in 1925 and never returned. In addition to the log, there is a long list of jewelry and other valuables held in the ship's safe. The ship sank when it collided with a freighter and the wreckage was never found. The log tells that the Hedonist was going due south at a speed of 20 knots in calm seas through a rare fog just before the collision. While in the fog, it was struck broadside by the 60,000-ton freighter, the Ironhorse, which was traveling west at 10 knots. The log tells the exact location (latitude and longitude) of the liner just before the collision. Of course, your employer is keeping that information secret for now. The log also notes that when the freighter's bow pierced the hull of the liner, the two ships were stuck together and sank together. To get the summer job, you are asked to help determine the search area by calculating the velocity (magnitude and direction) of the ships just after collision.
18. You have been hired to check the technical correctness of an upcoming made-for-TV murder mystery. The mystery takes place in the space shuttle. In one scene, an astronaut's safety line is sabotaged while she is on a space walk, so she is no longer connected to the space shuttle. She checks and finds that her thruster pack has also been damaged and no longer works. She is 200 meters from the shuttle and moving with it. That is, she is not moving with respect to the shuttle. There she is drifting in space with only 4 minutes of air remaining. To get back to the shuttle, she decides to unstrap her 10-kg tool kit and throw it away with all her strength, so that it has a speed of 8 m/s. In the script, she survives, but is this correct? Her mass, including space suit, is 80 kg.

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19. You have been hired as a technical consultant for an early-morning cartoon series for children to make sure that the science is correct. In the script, a wagon containing two boxes of gold (total mass of 150 kg) has been cut loose from the horses by an outlaw. The wagon starts from rest 50 meters up a hill with a 6° slope. The outlaw plans to have the wagon roll down the hill and across the level ground and then crash into a canyon where his confederates wait. But in a tree 40 meters from the edge of the canyon wait the Lone Ranger (mass 80 kg) and Tonto (mass 70 kg). They

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drop vertically into the wagon as it passes beneath them. The script states that it takes the Lone Ranger and Tonto 5 seconds to grab the gold and jump out of the wagon, but is this correct?. You assume that the wagon rolls with negligible friction.

20. You are helping your friend prepare for her next skate board exhibition. For her program, she plans to take a running start and then jump onto her heavy duty 15-lb stationary skateboard. She and the skateboard will glide in a straight line along a short, level section of track, then up a sloped concrete wall. She wants to reach a height of at least 10 feet above where she started before she turns to come back down the slope. She has measured her maximum running speed to safely jump on the skateboard at 7 feet/second. She knows you have taken physics, so she wants you to determine if she can carry out her program as planned. She tells you that she weighs 100 lbs.
21. Because of your physics background, you have been hired as a technical advisor for a new James Bond adventure movie. In the script, Bond and his latest love interest, who is $\frac{2}{3}$ his weight (including skis, boots, clothes, and various hidden weapons), are skiing in the Swiss Alps. She skis down a slope while he stays at the top to adjust his boot. When she has skied down a vertical distance of 100 ft, she stops to wait for him and is captured by the bad guys. Bond looks up and sees what is happening. He notices that she is standing with her skis pointed downhill while she rests on her poles. To make as little noise as possible, Bond starts from rest and glides down the slope heading right at her. Just before they collide, she sees him coming and lets go of her poles. He grabs her and they both continue downhill together. At the bottom of the hill, another slope goes uphill and they continue to glide up that slope until they reach the top of the hill and are safe. The writers want you to calculate the maximum possible height that the second hill can be relative to the position where the collision took place. Both Bond and his girl friend are using new, top-secret frictionless stealth skis developed for the British Secret Service.
22. Because of your concern that incorrect science is being taught to children when they watch cartoons on TV, you have joined a committee which is reviewing a new cartoon version of Tarzan. In this episode, Tarzan is on the ground in front of a herd of stampeding elephants. Just in time Jane, who is up in a tall tree, sees him. She grabs a convenient vine and swings towards Tarzan, who has twice her mass, to save him. Luckily, the lowest point of her swing is just where Tarzan is standing. When she reaches him, he grabs her and the vine. They both continue to swing to safety over the elephants up to a height which looks to be about $\frac{1}{2}$ that of Jane's original position. To decide if you going to approve this cartoon, calculate the maximum height Tarzan and Jane can swing as a fraction of her initial height.
23. You are watching a Saturday morning cartoon concerning a jungle hero called George of the Jungle. George attempts to save his friend, an ape named Ape, from a stampeding herd of wildebeests. Ape is at the base of a tall tree which has a vine attached to its top. George is in another tree holding the other end of the vine. George plans to swing down from the tree, grab Ape at the bottom of the swing, and continue up to safety on a ledge which is half of George's initial height in the tree. Assuming that Ape weighs the same as George, will they successfully make it to the top of the ledge?

24. Your friend has just been in a traffic accident and is trying to negotiate with the insurance company of the other driver to pay for fixing her car. She believes that the other car was speeding and therefore the accident was the other driver's fault. She knows that you have a knowledge of physics and hopes that you can prove her conjecture. She takes you out to the scene of the crash and describes what happened. She was traveling North when she entered the fateful intersection. There was no stop sign, so she looked in both directions and did not see another car approaching. It was a bright, sunny, clear day. When she reached the center of the intersection, her car was struck by the other car which was traveling East. The two cars remained joined together after the collision and skidded to a stop. The speed limit on both roads entering the intersection is 50 mph. From the skid marks still visible on the street, you determine that after the collision the cars skidded 56 feet at an angle of 30° north of east before stopping. She has a copy of the police report which gives the make and year of each car. At the library you determine that the weight of her car was 2600 lbs. and that of the other car was 2200 lbs., where you included the driver's weight in each case. The coefficient of kinetic friction for a rubber tire skidding on dry pavement is 0.80. It is not enough to prove that the other driver was speeding to convince the insurance company. She must also show that she was under the speed limit.
25. Because movie producers have come under pressure for teaching children incorrect science, you have been appointed to help a committee of concerned parents review a script for a new Superman movie. In the scene under consideration, Superman rushes to save Lois Lane who has been pushed from a window 300 feet above a crowded street. Superman is 0.5 miles away when he hears Lois scream and rushes to save her. He swoops down in the nick of time, arriving when Lois is just 3.0 feet above the street, and stopping her just at ground level. Lois changes her expression from one of horror at her impending doom to a smile of gratitude as she gently floats to the ground in Superman's arms. The committee wants to know if there is really enough time to express this range of emotions, even if there is a possible academy award on the line. The chairman asks you to calculate the time it takes for Superman to stop Lois's fall. To do the calculation, you assume that Superman applies a constant force to Lois in breaking her fall and that she weighs 120 lbs. While thinking about this scene you also wonder if Lois could survive the force that Superman applies to her.
26. This year you have a summer job working for the National Park Service. Since they know that you have taken physics, they start you off in the laboratory which tests possible new equipment. Your first job is to test a small cannon. During the winter, small cannons are used to prevent avalanches in populated areas by shooting down heavy snow concentrations overhanging the sides of mountains. In order to determine the range of the cannon, it is necessary to know the velocity with which the projectile leaves the cannon (muzzle velocity). The cannon you are testing has a weight of 700 lbs. and shoots a 40-lb projectile. During the lab tests the cannon is held horizontally in a rigid support so that it cannot move. Under those conditions, you measure the magnitude of the muzzle velocity to be 400 m/s. When the cannon is actually used in the field, however, it is mounted so that it is free to move (recoil) when it is fired. Your boss asks you to calculate the projectile's speed leaving the cannon under field conditions, when it is allowed to recoil. She tells you to take the case where the cannon is fired horizontally using cannon shells which are identical to those used in the laboratory test.

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27. You have been able to get a part time job with a medical physics group investigating ways to treat inoperable brain cancer. One form of cancer therapy being studied uses slow neutrons to knock a particle (either a neutron or a proton) out of the nucleus of the atoms which make up cancer cells. The neutron knocks out the particle it collides with in an inelastic collision. The heavy nucleus essentially does not move in the collision. After a single proton or neutron is knocked out of the nucleus, the nucleus decays, killing the cancer cell. To test this idea, your research group decides to measure the change of internal energy of a nitrogen nucleus after a neutron collides with one of the neutrons in its nucleus and knocks it out. In the experiment, one neutron goes into the nucleus with a speed of 2.0×10^7 m/s and you detect two neutrons coming out at angles of 30° and 15° . You can now calculate the change of internal energy of the nucleus.

Additional Conservation Problems: