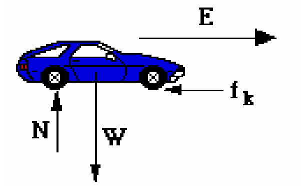
Teaching Introductory Physics Through Problem Solving

"I understand the material, I just can't solve the problems."



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15 year continuing effort to improve undergraduate education with contributions by:
Many faculty and graduate students of U of M Physics & Education
In collaboration with U of M Physics Education Group

Details at http://groups.physics.umn.edu/physed/

Supported in part by NSF, FIPSE and the University of Minnesota

Key Features of the Course

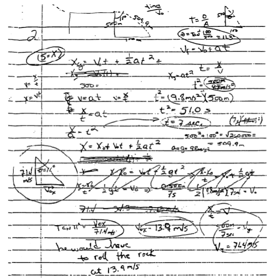
Connect all concepts to students' reality

- Everything in a context
- Everything motivated
  - Solving a problem is a motivation
  - Context-rich problems

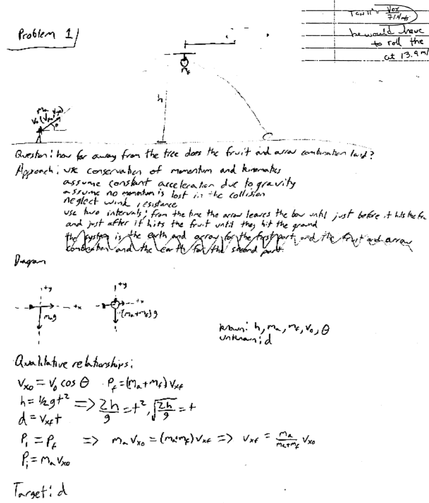
•Stress practice and repetition without boredom

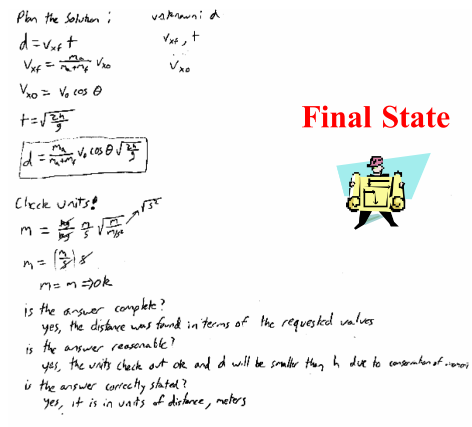
- Teach a general framework for solving all problems
- Use cooperative group work to facilitate learning
  - Peer coaching
  - Guided practice

Student Problem Solutions



Initial State





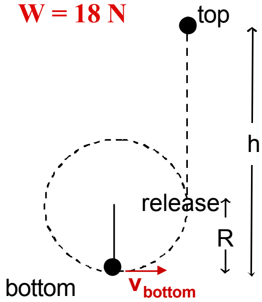
Final State

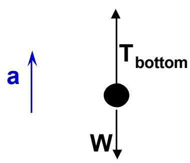
Problem Used in the Interview

You are whirling a stone tied to the end of a string around in a vertical circle having a radius of 65 cm. You wish to whirl the stone fast enough so that when it is released at the point where the stone is moving directly upward it will rise to a **maximum height of 23 meters** above the lowest point in the circle. In order to do this, **what force will you have to exert on the string** when the stone passes through its lowest point one-quarter turn before release? Assume that by the time that you have gotten the stone going and it makes its final turn around the circle, you are holding the end of the string at a fixed position. **Assume also that air resistance can be neglected.** The stone weighs 18 N.

Final examination question (Fall, 1997)

R = 0.65 m
h = 23 m
W = 18 N





Free body diagram at bottom

An Expert Solution

- No work is done by string (since $T \perp v$),
- so all work is done by gravity. Using conservation of energy between bottom and top:

$$\frac{1}{2}mv_{bottom}^2 = mgh$$

Using Newton's 2nd Law at the bottom.

$$T_{bottom} - W = m\frac{v_{bottom}^2}{R}$$

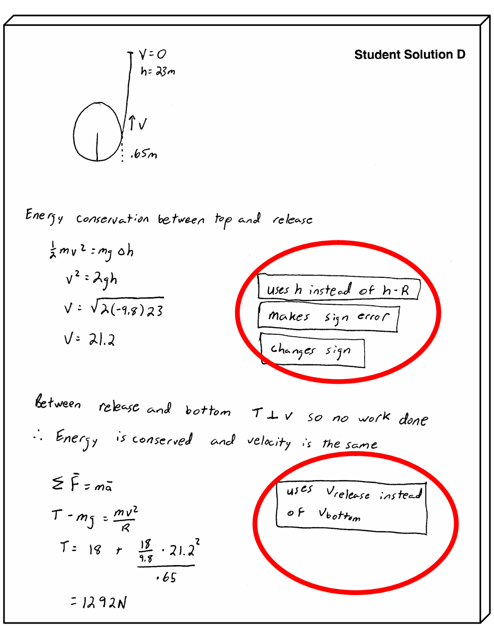
$$T_{bottom} - W = \frac{2mgh}{R}$$

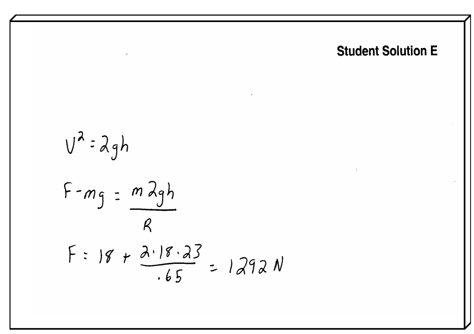
$$T_{bottom} - W = \frac{2Wh}{R}$$

$$T_{bottom} = W\left(\frac{2h}{R} + 1\right)$$

$$T_{bottom} = 1292\text{ N}$$

Student Solutions D and E

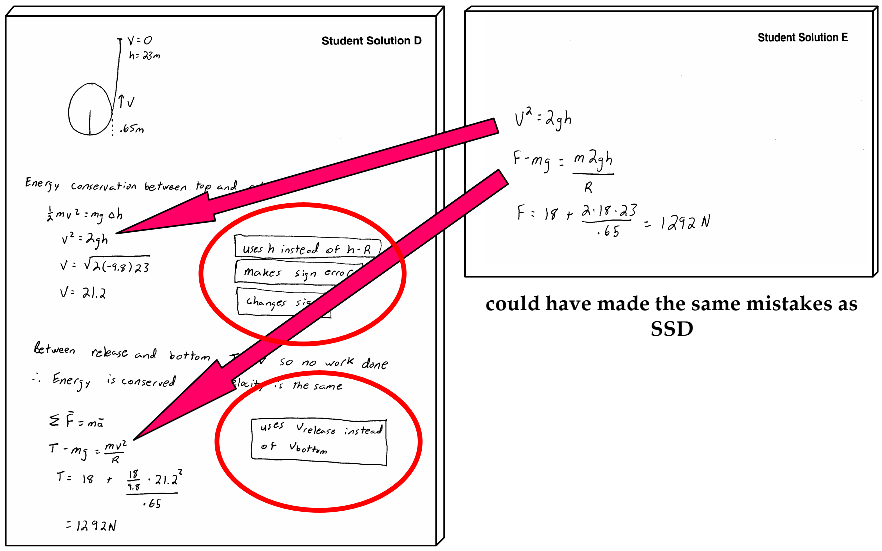




comments made by interviewers

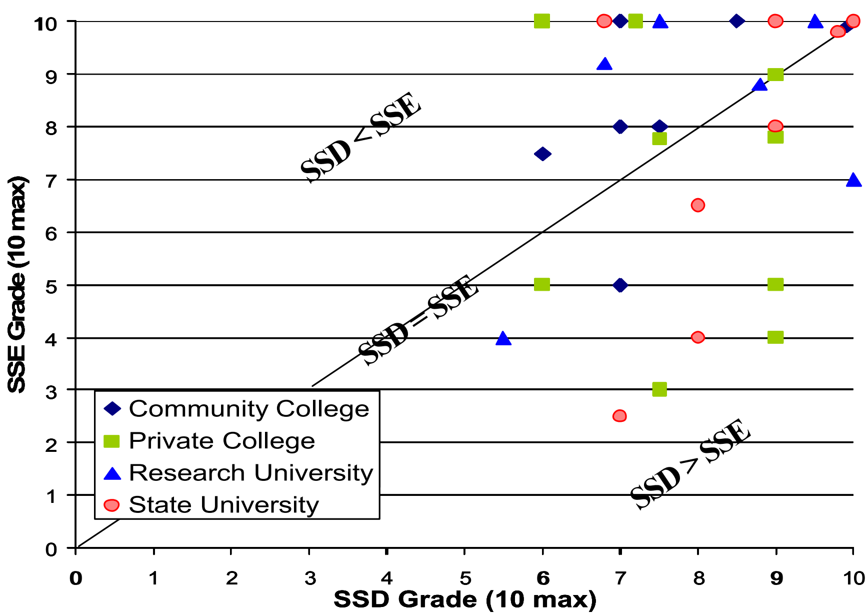
grade these solutions on a 10-point scale

Student Solutions D and E



comments made by interviewers

How did Interviewees Grade?



Looking at faculty from RU

5 (out of 6) of the instructors expressed conflicting values when grading Student Solution E (short solution).

- **Value 1: Instructors want to see student reasoning so they can know if a student really understands.**
  - **"There's not a single word to tell you that he put these things down and didn't guess." (Instructor 4)**

Looking at faculty from RU

5 (out of 6) of the instructors expressed conflicting values when grading Student Solution E (short solution).

- **Value 1: Instructors want to see student reasoning so they can know if a student really understands.**

  - **Burden of Proof on Students**
- **Value 2: Instructors are reluctant to penalize a student who *might* be correct.**
  - **Burden of Proof on Instructors**
  - **"There's nothing in here that's wrong. Yeah, it's not clear what v is in $v^2=2gh$, but in the end the equation would come out the same." (Instructor 5: 10 pts.)**

Looking at faculty from RU

5 (out of 6) of the instructors expressed conflicting values when grading Student Solution E (short solution).

- **Value 1: Instructors want to see student reasoning so they can know if a student really understands.**
  - ❖ Burden of Proof on Students
- **Value 2: Instructors are reluctant to penalize a student who *might* be correct.**
  - ❖ Burden of Proof on Instructors
  - ❖ Viewing solution in best possible light:
    - ➢ "He had to know the 3 principles involved in the problem perfectly. Just had to." (Instructor 4: 7 pts.)

Resolving the Conflict

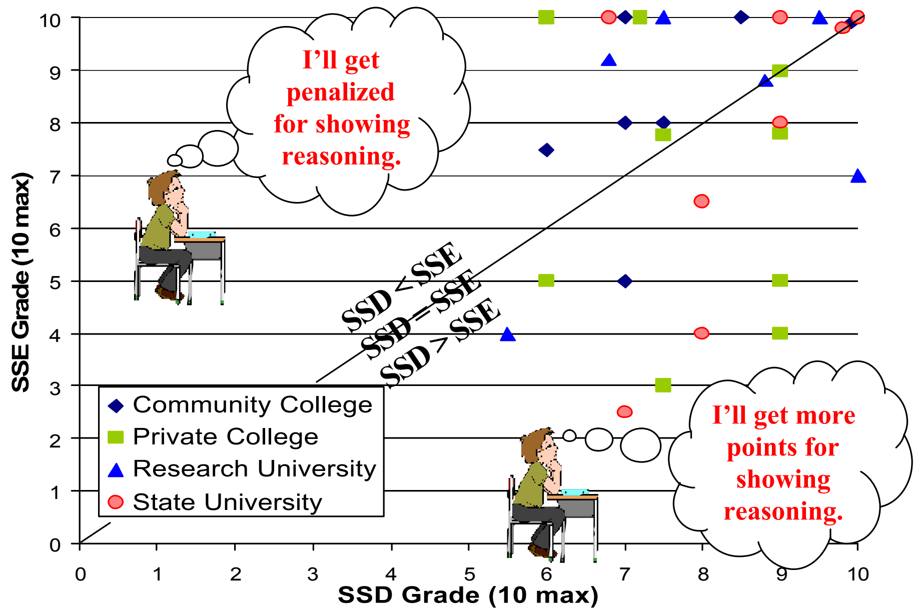
- **Value 1: Instructors want to see student reasoning so they can know if a student really understands.**

Value 2: Instructors are reluctant to penalize a student who *might* be correct.

←——————————————————————————→

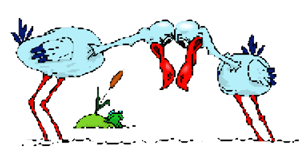
Insist on Reasoning		Compromise	Give Full Credit
Instructor 1 6 point penalty	Instructor 4 3 point penalty	Instructor 3 ~1 point penalty	Instructor 5 No Penalty
		Instructor 6 1 point penalty	Instructor 2 No Conflict No Penalty

What Message Is Sent to Students?



What can we say about the preliminary results?

Many physics instructors hold conflicting values when grading



- value seeing student reasoning in problem solutions

- yet many actually penalize students for showing reasoning

What Is Problem Solving?

"Process of Moving Toward a Goal When Path is Uncertain"

- **If you know how to do it, its not a problem.**

Problems are solved using tools

General-Purpose Heuristics

Not algorithms

"Problem Solving Involves Error and Uncertainty"

A problem for your student is not a problem for you

Exercise vs Problem

M. Martinez, Phi Delta Kappan, April, 1998

Some Heuristics

Means - Ends Analysis

identifying goals and subgoals

Working Backwards

step by step planning from desired result

Successive Approximations

range of applicability and evaluation

External Representations

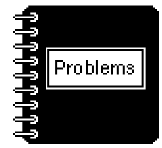
pictures, diagrams, mathematics

General Principles of Physics

Students' Misconceptions About Problem Solving

You need to know the right formula to solve a problem:

- Memorize formulas
- Memorize solution patterns



Actions that reinforce the misconception

- Test requires students to remember important equations
- Allow students to bring in "crib" sheets

It's all in the mathematics:

- Manipulate the equations as quickly as possible
    - Plug-and-chug
- Numbers are easier to deal with
    - Plug in numbers as soon as possible

$$\sum F = ma$$
$$T - f = ma$$
$$a = \frac{T - f}{m}$$

Actions that reinforce the misconception

- Single step problems.
- Multi-part problems.

The Monotillation of Traxoline

(attributed to Judy Lanier)

It is very important that you learn about traxoline. Traxoline is a new form of zionter. It is montilled in Ceristanna. The Ceristannians gristerlate large amounts of fevon and then brachter it to quasel traxoline. Traxoline may well be one of our most lukized snezlaus in the future because of our zionter lescelidge.

Answer the following questions.

1. What is traxoline?
2. Where is traxoline montilled?
3. How is traxoline quasselled?
4. Why is it important to know about traxoline?

A Complex Process

The procedure is quite simple. First you arrange them into different groups. Of course, one group may be sufficient depending on how much there is to do. You may have to go somewhere else due to lack of facilities.

Next you actually accomplish your goal. But a mistake can be expensive. It is important not to overdo things. It is usually better to do too few things than too many. This is especially important when issues of compatibility arise. At first, the whole procedure might seem complicated since timing can be crucial. In the immediate future it is unlikely that the need for this process will diminish, but then, one can never tell.

After the procedure is completed, one forms different groups again. Then things can be put into their appropriate places. Every so often the whole cycle will then need to be repeated. However, that is a part of life.

Answer the following questions.

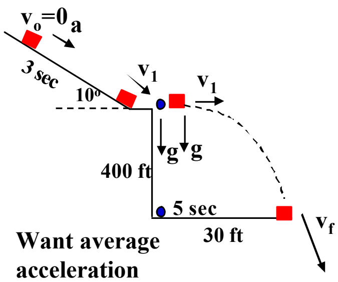
1. **What is the process being discussed?**
2. **What facilities are needed?**
3. **What are some compatibility issues?**
4. **Why is it important to form groups?**

Laundry

Context-Rich Problem

You have a summer job with an insurance company and are helping to investigate a tragic "accident." At the scene, you see a road running straight down a hill that is at 10° to the horizontal. At the bottom of the hill, the road widens into a small, level parking lot overlooking a cliff. The cliff has a vertical drop of 400 feet to the horizontal ground below where a car is wrecked 30 feet from the base of the cliff. A witness claims that the car was parked on the hill and began coasting down the road, taking about 3 seconds to get down the hill. Your boss drops a stone from the edge of the cliff and, from the sound of it hitting the ground below, determines that it takes 5.0 seconds to fall to the bottom. You are told to calculate the car's average acceleration coming down the hill based on the statement of the witness and the other facts in the case. Obviously, your boss suspects foul play.

Visualize



Want average acceleration down the hill.

Principles

- Average acceleration = (velocity change / time for change)
- Final velocity = initial horizontal velocity of flight
- Vertical & horizontal motion are independent

In flight

- Horizontal velocity is constant
- Vertical acceleration is constant & same for everything

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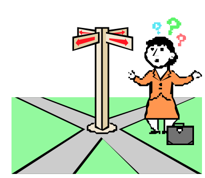
Teaching Students to Solve Problems

Solving Problems Requires Conceptual Knowledge:

From Situations to Decisions

- Visualize situation
- Determine goal
- Choose applicable principles
- Choose relevant information
- Construct a plan
- Arrive at an answer
- Evaluate the solution



Students must be taught *explicitly*

The difficulty -- major misconceptions, lack of metacognitive skills, no heuristics

Problem Solving Requires

Metacognative Skills

- Managing time and direction
- Determining next step
- Monitoring understanding
- Asking skeptical questions
- Reflecting on own learning process

Practice Makes Perfect
BUT

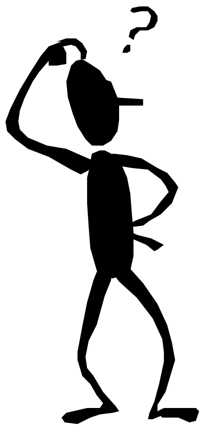
Traditional "Problems"

- Can often be solved by manipulating equations
- Little visualization necessary
- Few decisions necessary
- Disconnected from student's reality
- Can often be solved without knowing physics

What is being practiced?

Problem that Does Not Support Problem Solving

A block starts from rest and accelerates for 3.0 seconds. It then goes 30 ft. in 5.0 seconds at a constant velocity.



a. What was the final velocity of the block?

b. What was the acceleration of the block?

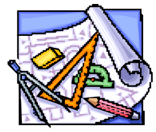
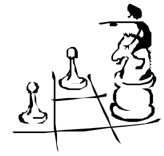
Textbook Problem

Appropriate Problems for Problem Solving

The problems must be challenging enough so there is a *real* advantage to using problem solving heuristics.

1. **The problem must be complex enough so the best student in the class is not certain how to solve it.**

   The problem must be simple enough so that the solution, once arrived at, can be understood and appreciated.

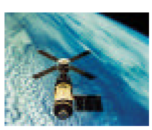
2. The task must be designed so that

   • the major problem solving heuristics are required (e.g. physics understood, a situation requiring an external representation);

   • there are several decisions to make in order to do the task (e.g. several different quantities that could be calculated to answer the question; several ways to approach the problem);

   • the task cannot be resolved in a few steps by copying a pattern.

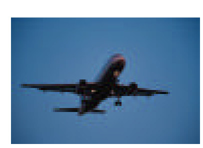




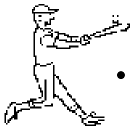

3. The task problem must connect to each student's mental processes

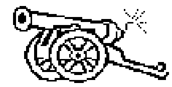
   • the situation is real to the student so other information is connected;

   • there is a reasonable goal on which to base decision making.

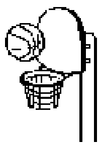



Context-rich Problems

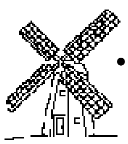




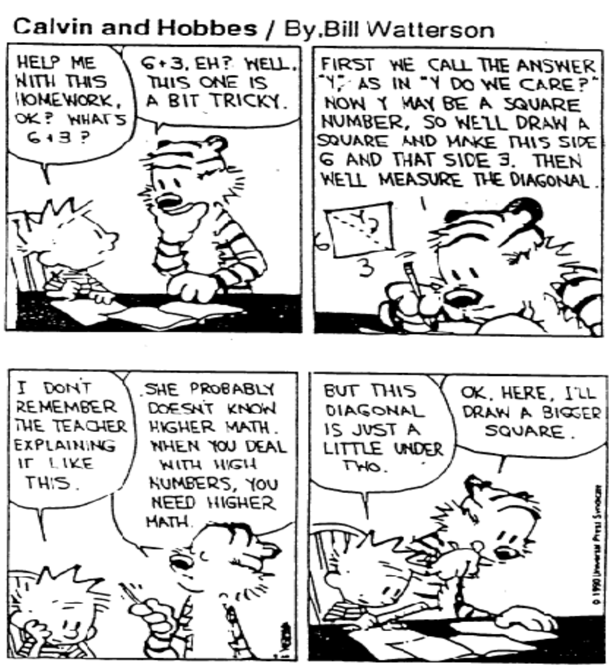
- **Each problem is a short story in which the major character is the student. That is, each problem statement uses the personal pronoun "you."**
- **Some decisions are necessary to proceed.**



- **The problem statement includes a plausible motivation or reason for "you" to calculate something.**
- **The objects in the problems are real (or can be imagined) -- the idealization process occurs explicitly.**



- **No pictures or diagrams are given with the problems. Students must visualize the situation by using their own experiences.**
- **The problem can not be solved in one step by plugging numbers into a formula.**

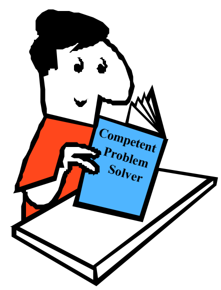


Problem-solving Framework
Used by experts in all fields

STEP 1 — **Recognize the Problem**
What's going on?

STEP 2 — **Describe the problem in terms of the field**
What does this have to do with ...... ?

STEP 3 — **Plan a solution**
How do I get out of this?



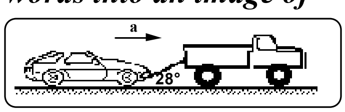
STEP 4 — **Execute the plan**
Let's get an answer

STEP 5 — **Evaluate the solution**
Can this be true?

Step	Bridge

1. Focus on the Problem

Translate the words into an image of the situation.

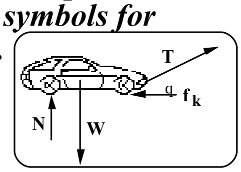


Identify an approach to the problem.

Relate forces on car to acceleration using Newton's Second Law

2. Describe the Physics

Translate the mental image into a physics representation of the problem (e.g., idealized diagram, symbols for knowns and unknowns).



Assemble mathematical tools (equations).

$\sum F = ma$

$f_k = \mu N$

$W = mg$

3. Plan a Solution

Step — **Bridge**

3. Plan a Solution

Translate the physics description into a mathematical representation of the problem.

Find a:
[1] $\sum F_x = ma_x$
Find $\sum F_x$:
[2] $\sum F_x = T_x - f_k$

Outline the mathematical solution steps.

Solve[3] for T_x and put into [2].
Solve[2] for $\sum F_x$ and put into [1].
Solve[1] for a_x.

4. Execute the Plan

Translate the plan into a series of appropriate mathematical actions.

$$T_x - f_k = ma_x$$

$$T\cos\theta - \mu(W - T\sin\theta) = \frac{W}{g}a_x$$

$$\frac{gT}{W}(\cos\theta - \mu\sin\theta) - \mu g = a_x$$

Check units of algebraic solution.

$$\frac{\left[\frac{m}{s^2}\right][N]}{[N]} - \left[\frac{m}{s^2}\right] = \left[\frac{m}{s^2}\right] \quad \text{OK}$$

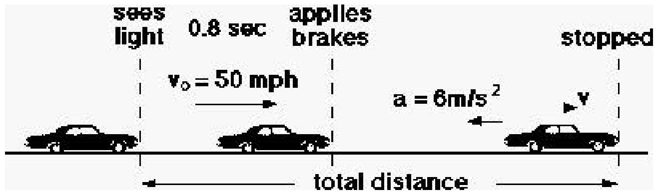
5. Evaluate the Solution

A Problem

You are driving on a freeway following another car when you wonder what your stopping distance would be if that car jammed on its brakes. You are going at 50 mph. When you get home you decide to do the calculation. You measure your reaction time to be 0.8 seconds from the time you see the car's brake lights until you apply your own brakes. Your owner's manual says that your car slows down at a rate of 6 m/s^2 when the brakes are applied.

Focus on the Problem

Picture and Given Information:



Question:

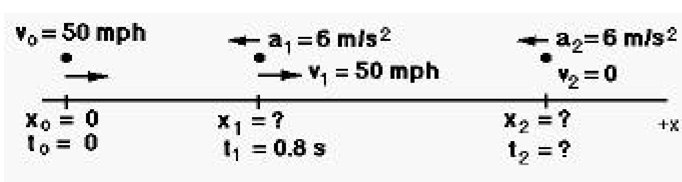
What total distance did the car travel to stop?

Approach:

- **The velocity is constant until brakes applied, then the acceleration is constant.**
- **Use the definition of velocity and acceleration.**

Describe the Physics

Diagram and Define Physics Quantities:



Target Quantity(s): **Find x_2**

Quantitative Relationships:

From 0 to 1:

$$v_{av1} = \frac{x_1 - x_0}{t_1 - t_0} = \frac{x_1}{t_1}$$

$$v_0 = v_1 = v_{av1} = v$$

From 1 to 2:

$$v_{av2} = \frac{v_1 + v_2}{2} = \frac{v}{2}$$

$$v_{av2} = \frac{x_2 - x_1}{t_2 - t_1}$$

$$a_{av} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{-v}{t_2 - t_1}$$

$$a_1 = a_2 = a_{av} = a$$

Plan the Solution

Construct Specific Equations:

Unknowns

Find x_2 — Unknowns: x_2

① $v_{av2} = \dfrac{x_2 - x_1}{t_2 - t_1}$ — Unknowns: v_{av2}, x_1, t_2

Find v_{av2}

② $v_{av2} = \dfrac{v}{2}$

Find x_1:

③ $v_{av1} = \dfrac{x_1 - x_0}{t_1 - t_0} = \dfrac{x_1}{t_1}$

Find t_2

④ $a = \dfrac{-v}{t_2 - t_1}$

Check for sufficiency:

Four unknowns
(x_2, v_{av2}, x_1, t_2)
Four equations.

Outline math solution:

Solve ④ for t_2,
put into ①.

Solve ③ for x_1,
put into ① .

Solve ② for v_{av2},
put into ① .

Solve ① for x_2.

Execute the Plan

Follow the Plan:

Solve ④ for t_2

$$a = \frac{-v}{t_2 - t_1}$$

$$at_2 - at_1 = -v$$

$$t_2 = \frac{at_1 - v}{a}$$

$$t_2 = t_1 - \frac{v}{a}$$

Solve ③ for x_1

$$v = \frac{x_1}{t_1}$$

$$x_1 = vt_1$$

Solve ② for v_{av2}

$$v_{av2} = \frac{v}{2}$$

Put all into ①

$$v_{av2} = \frac{x_2 - x_1}{t_2 - t_1}$$

$$\frac{v}{2} = \frac{x_2 - vt_1}{t_1 - \frac{v}{a} - t_1}$$

$$\frac{v}{2} = \frac{x_2 - vt_1}{-\frac{v}{a}}$$

$$-\frac{v^2}{2a} = x_2 - vt_1$$

$$vt_1 - \frac{v^2}{2a} = x_2$$

Calculate Target Quantity:

$$x_2 = (22.4\,\text{m/s})(0.8\,\text{s}) - \frac{(22.4\,\text{m/s})^2}{2(-6\,\text{m/s}^2)}$$

$$= 18\ \text{m} + 42\ \text{m}$$

$$= 60\ \text{m}$$

Evaluate the Answer

Is the Answer properly stated?

- **Yes. The total distance traveled by car to stop has been calculated.**

x_2 is in the units of length

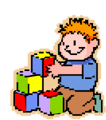
$$x_2 = \left(\frac{\mathbf{m}}{\mathbf{s}}\right)\mathbf{s} + \left[\frac{\left(\frac{\mathbf{m}}{\mathbf{s}}\right)^2}{\frac{\mathbf{m}}{\mathbf{s}^2}}\right]$$

$$= \mathbf{m} + \mathbf{m}$$

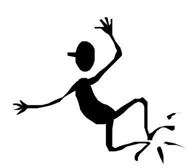
Is the Answer unreasonable?

No. A car length is about 6 m so 10 car lengths is not unreasonable.

The Dilemma



Start with simple problems to learn expert-like framework.



Success using novice strategies.

Why change?

Start with complex problems so novice strategy fails

Difficulty using new framework.

Why change?

What Using Cooperative Groups Does for Teaching Problem Solving

1. Following a problem solving framework seems too long and complex for most students.

   Cooperative-group problem solving allows practice until the framework becomes more natural.

2. Complex problems that need a strategy are initially difficult.

   Groups can successfully solve them so students see the advantage of a logical problem-solving framework early in the course.

What Using Cooperative Groups Does for Teaching Problem Solving

3. The group interactions externalize the planning and monitoring skills needed to solve problems allowing students to observe them. (Metacognition)

4. Students practice using the language of the field -- "talking physics."

5. Students must deal with and resolve their misconceptions.

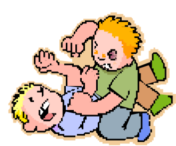
6. Coaching by instructors is more effective

   External clues of group difficulties
   Group processing of instructor input

Cooperative Groups

- Positive Interdependence
- Face-to-Face Interaction
- Individual Accountability
- Explicit Collaborative Skills
- Group Functioning Assessment

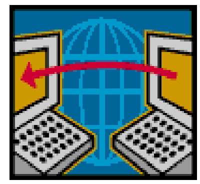
Why Group Problem Solving May Not Work



1. Inappropriate Tasks
2. Inappropriate Grading
3. Poor structure and management of Groups

The End

Please visit our website for more information:



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