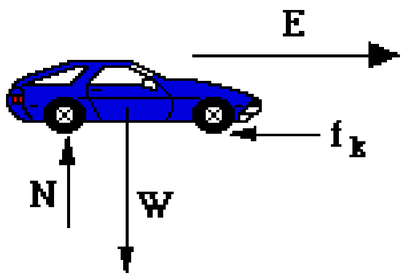
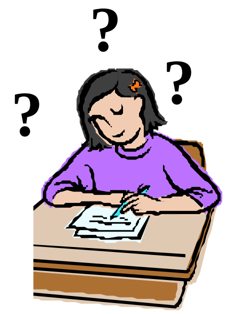
Teaching Introductory Physical Science Through Problem Solving

"I understand the material, I just can't solve the problems."



$$\sum F = ma$$
$$f_k = \mu N$$
$$W = mg$$



Ken Heller
School of Physics and Astronomy
University of Minnesota

15 year continuing project to improve undergraduate education with contributions by:
Many faculty and graduate students of U of M Physics & Education
In collaboration with U of M Physics Education Group

Pat Heller, Leon Hsu, Paul Knutson, Tom Thaden-Koch – University of Minnesota

Vince Kuo – North Carolina State University
Charles Henderson– Western Michigan University
Edit Yerushalmi – Weizmann Institute
Laura McCullough – University of Wisconsin, Stout
Tom Foster – University of Southern Illinois
Jennifer Blue – Miami University, Ohio
Mark Hollabaugh -Normandale Community College
Ron Keith – Emporia State University

Details at http://groups.physics.umn.edu/physed/

Supported in part by NSF, FIPSE and the University of Minnesota

Research Based Curriculum and Assessment

Goals

- Why Solve Problems?
- What are Problems?
- Experts and Novices

Students

- Skills & Misconceptions

Teaching Problem Solving?

- Instructional Framework
- Supporting Problem Solving

Instructors

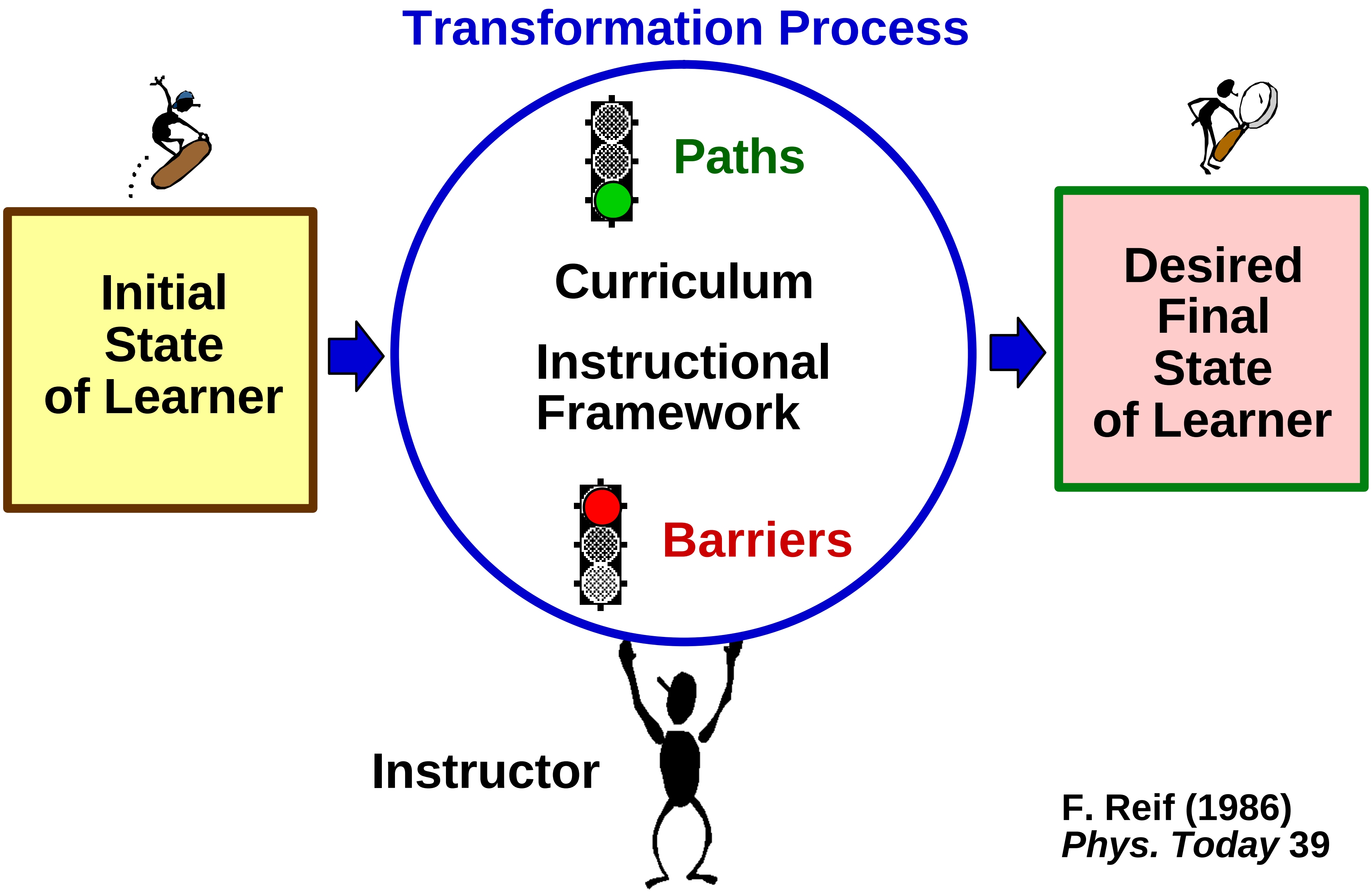
- Beliefs & Values

How Well Does It Work

- Data

Teaching

<final | T | initial>



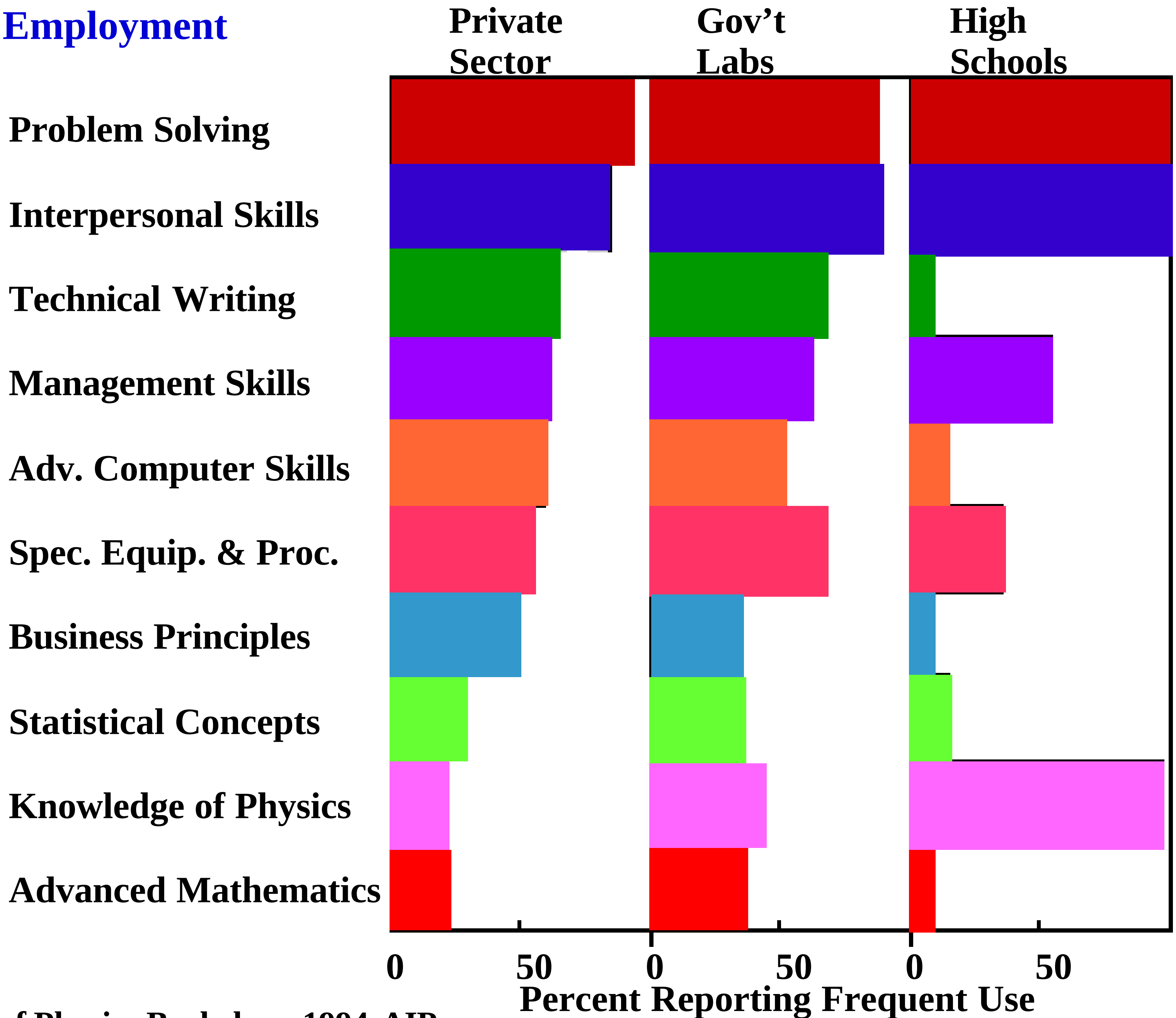
F. Reif (1986)
Phys. Today 39

Why Teach Students to Solve Problems?

- **Society Wants It**
- **Employers Want It**
- **Other Departments Want It**
- **Our Department Wants It**

What is IT?

Employment



Survey of Physics Bachelors, 1994-AIP

What Do Other Faculty Want?

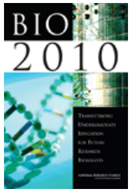
Goals: Calculus-based Course (88% engineering majors) 1993

4.5	Basic principles behind all physics
4.5	General qualitative problem solving skills
4.4	General quantitative problem solving skills
4.2	Apply physics topics covered to new situations
4.2	*Use with confidence*

Goals: Algebra-based Course (24 different majors) 1987

4.7	Basic principles behind all physics
4.2	General qualitative problem solving skills
4.2	*Overcome misconceptions about physical world*
4.0	General quantitative problem solving skills
4.0	Apply physics topics covered to new situations

Goals: Biology Majors Course 2003



4.9	Basic principles behind all physics
4.4	General qualitative problem solving skills
4.3	*Use biological examples of physical principles*
4.2	*Overcome misconceptions about physical world*
4.1	General quantitative problem solving skills
4.0	*Apply physics topics covered to real world situations*
4.0	*Know range of applicability of physics principles*

Modified survey in response to CBS Curriculum Committee

Answer This Question

You have a summer job with an insurance company and are helping to investigate a tragic "accident." At the scene, you see a road running straight down a hill that is at 10° to the horizontal. At the bottom of the hill, the road widens into a small, level parking lot overlooking a cliff. The cliff has a vertical drop of 400 feet to the horizontal ground below where a car is wrecked 30 feet from the base of the cliff. A witness claims that the car was parked on the hill and began coasting down the road, taking about 3 seconds to get down the hill. Your boss drops a stone from the edge of the cliff and, from the sound of it hitting the ground below, determines that it takes 5.0 seconds to fall to the bottom. You are told to calculate the car's average acceleration coming down the hill based on the statement of the witness and the other facts in the case. Obviously, your boss suspects foul play.

Work in a group of 3 – Introduce yourselves & decide who will

- **Document your procedure.**
- **Document decisions you make.**
- **Document your solution.**

Time: 5 minutes

What Is Problem Solving?

Process of Moving Toward a Goal When Path is Uncertain

- **If you know how to do it, its not a problem.**

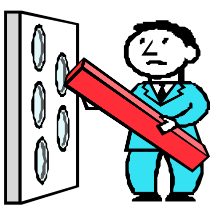
Problems are solved using tools

General-Purpose Heuristics

Not Algorithms, Not Patterns

Problem Solving Requires Metacognitive Skills

Problem Solving Involves Error and Uncertainty



A problem for your student is not a problem for you

Exercise vs Problem

M. Martinez, Phi Delta Kappan, April, 1998

http://www.gse.uci.edu/doehome/DeptInfo/Faculty/Martinez/problem_solving.html

Some Reflective Skills (Metacognition)

- **Managing time and direction**
- **Determining next step**
- **Monitoring understanding**
- **Asking skeptical questions**
- **Reflecting on own learning process**

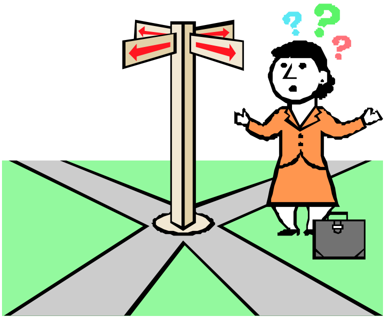
Some General Tools (Heuristics)

- **Means - Ends Analysis (identifying goals and subgoals)**
- **Working Backwards (step by step planning from desired result)**
- **Successive Approximations (idealization, approximation, evaluation)**
- **External Representations (pictures, diagrams, mathematics)**
- **General Principles of Physics (causality, interaction, conservation)**

Solving Problems is Making a Sequence of Decisions

Making Decisions Requires Conceptual Knowledge:

A Framework Organizes the Decisions



- How best to visualize the situation
- Determine goal
- Choose applicable principles
- Choose relevant information
- Construct a plan that gets to an answer
- Get the answer
- How can you evaluate the solution

Problem Solving is a Tool to Learn Concepts

Problem Solving is an End in Itself – Needs to be Taught in Context

Students must be taught *explicitly*

The difficulty -- major misconceptions, lack of metacognitive skills, no heuristics

Initial State of the Learner

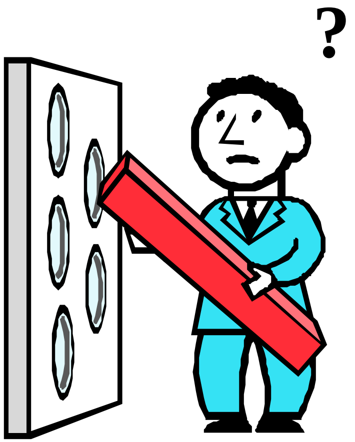
Students have Misconceptions about

The Field of Physics

Learning Physics

Nature

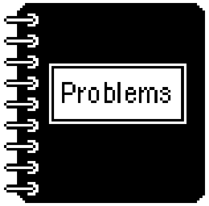
Problem-solving



All combine to make it difficult for students to solve problems.

Not the same as "getting a problem right".

Students' Misconceptions About Problem Solving



You need to know the right formula to solve a problem:

Memorize formulas

Memorize solution patterns

Bring in "crib" sheets

$$\sum F = ma$$
$$T - f = ma$$
$$a = \frac{T - f}{m}$$

It's all in the mathematics:

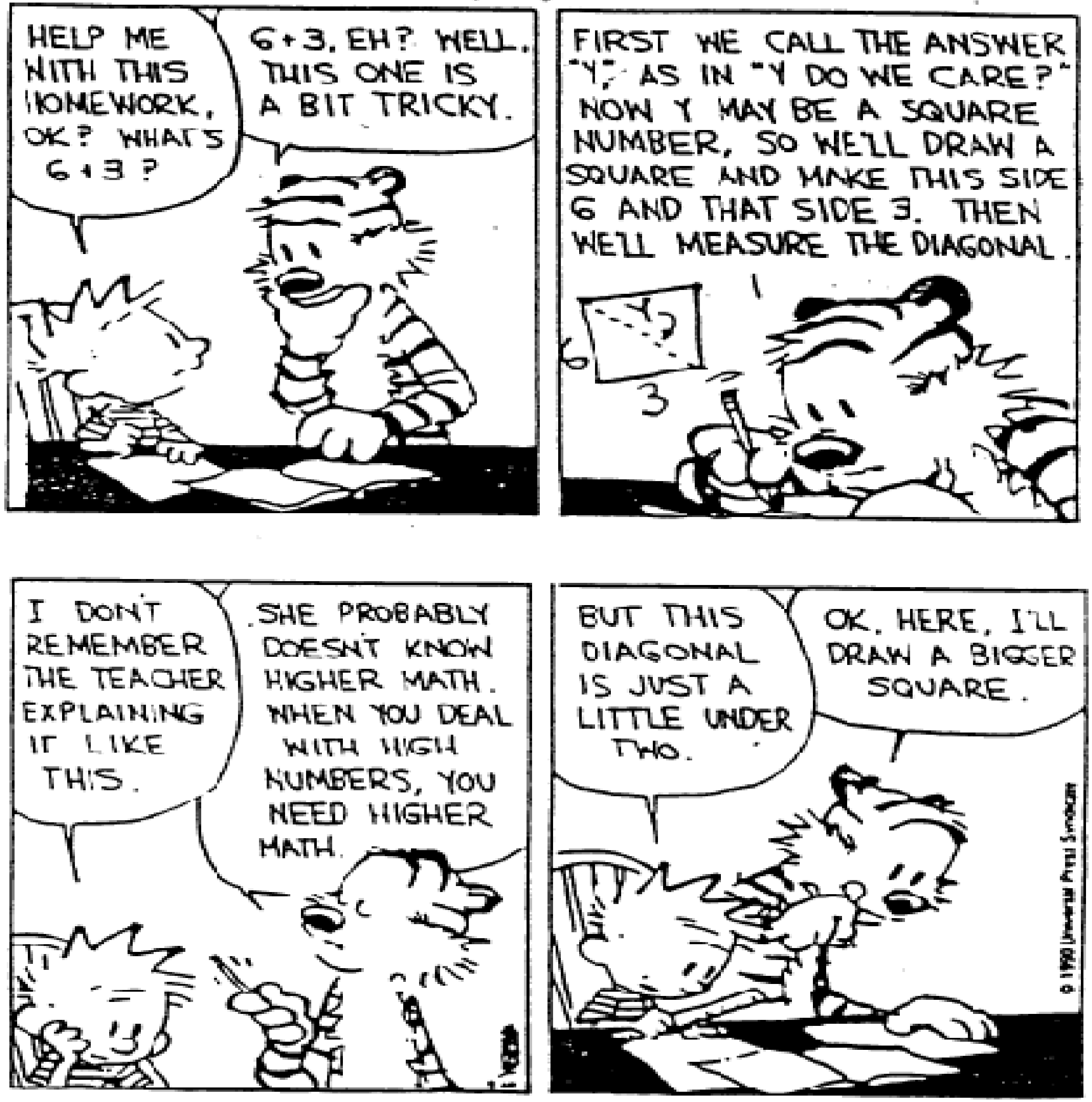
Manipulate the equations as quickly as possible

Plug-and-chug

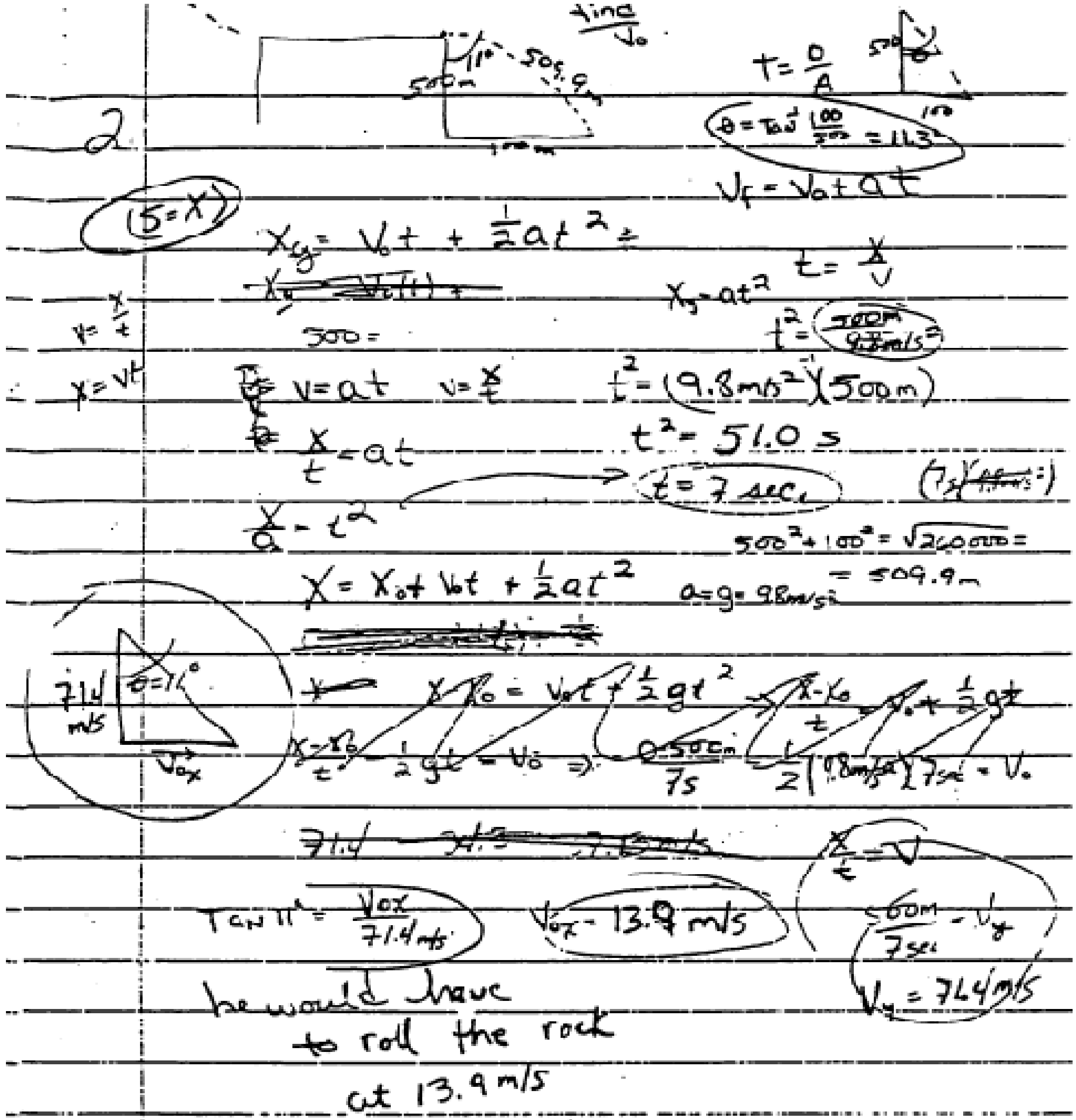
Numbers are easier to deal with

Plug in numbers as soon as possible

Calvin and Hobbes / By.Bill Watterson



Typical Student Test



The "Clear Explanation" Misconception

Commonly held by Faculty, TAs, Students, & Administrators

Instructor pours knowledge into students.

Little knowledge is retained.

Student's Fault

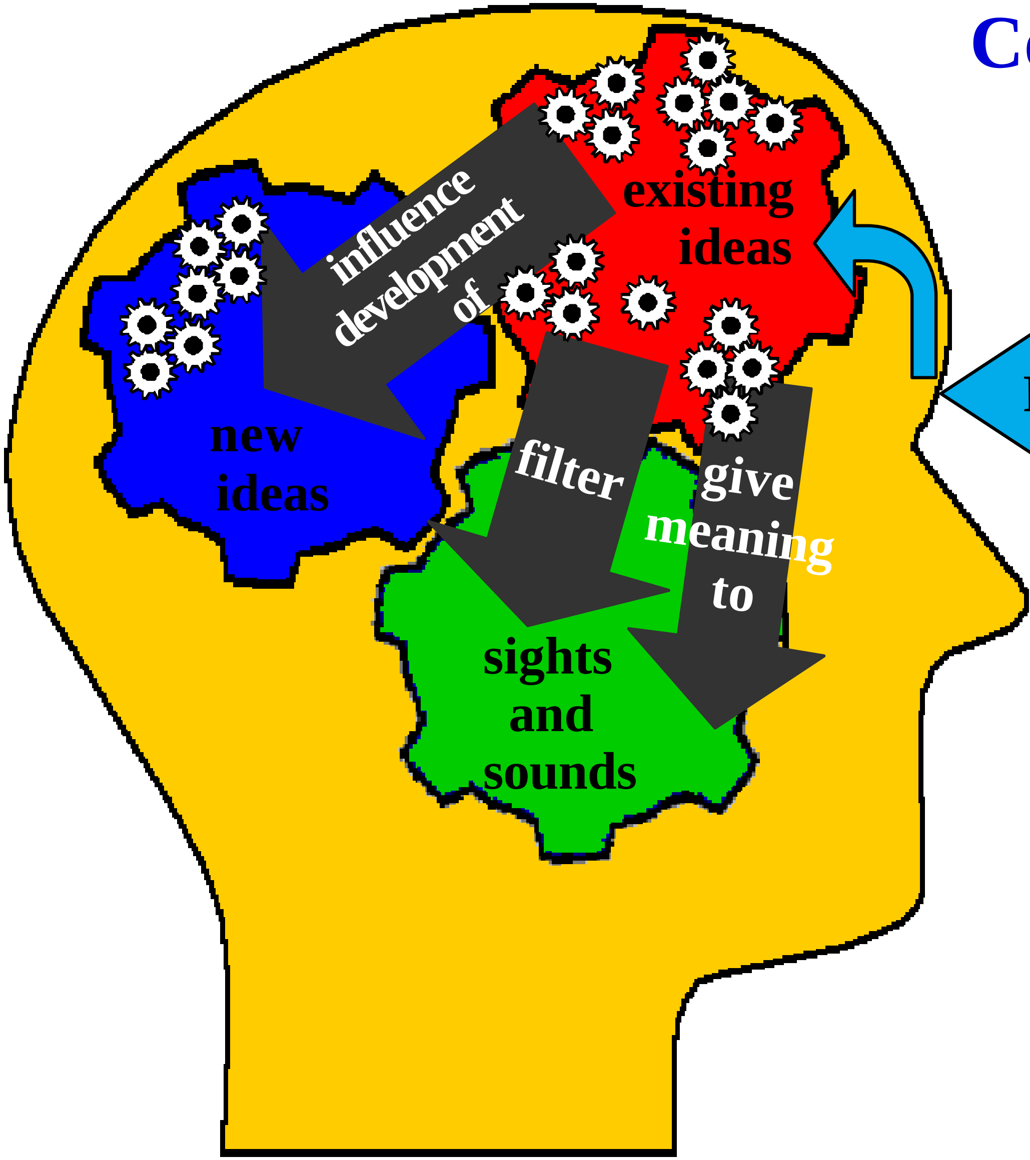
Impedance mismatch between student and instructor.

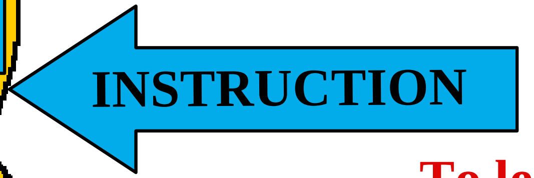
Instructor's Fault

Learning is more complicated

Leonard et. al. (1999). Concept-Based Problem Solving.

Cognitive Apprenticeship Instruction





Learning in the environment of expert practice

To learn golf
You have to see golf
You have to play golf

model

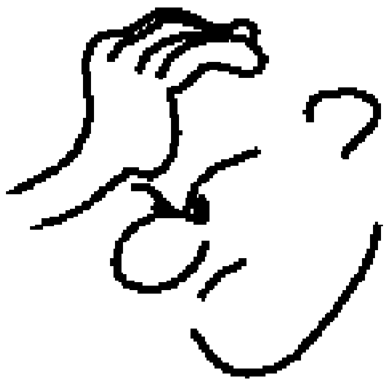
coach

fade

Collins, Brown, & Newman (1990)

Practice Makes Perfect

BUT



Traditional “Problems”

- Can often be solved by manipulating equations
- Little visualization necessary
- Few decisions necessary
- Disconnected from student’s reality
- Can often be solved without knowing physics

What is being practiced?

From a Textbook

Cart A, which is moving with a constant velocity of 3 m/s, has an inelastic collision with cart B, which is initially at rest as shown in Figure 8.3. After the collision, the carts move together up an inclined plane. Neglecting friction, determine the vertical height *h* of the carts before they reverse direction.

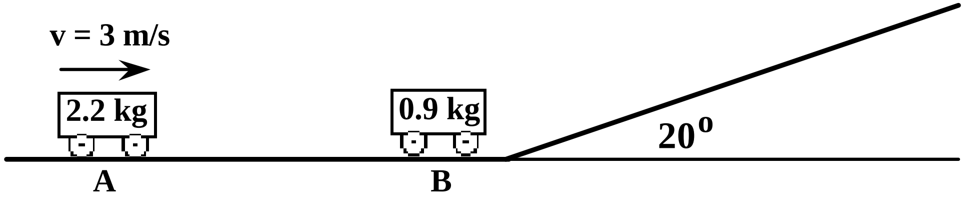


Figure 8.3

Novice Strategy

Read Problem

↓

Categorize problem by surface features

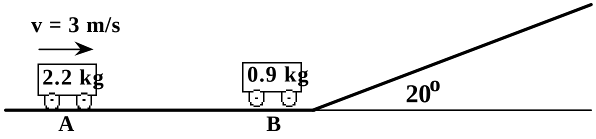
↓

Recall memorized pattern of actions and specific formulas for solving problem type

↓

Manipulate a procedure until solution obtained

Cart A, which is moving with a constant velocity of 3 m/s, has an inelastic collision with cart B, which is initially at rest as shown in Figure 8.3. After the collision, the carts move together up an inclined plane. Neglecting friction, determine the vertical height *h* of the carts before they reverse direction.



No context – "Y do we care?"
No decisions

The Monotillation of Traxoline

It is very important that you learn about traxoline. Traxoline is a new form of zionter. It is montilled in Ceristanna. The Ceristannians gristerlate large amounts of fevon and then brachter it to quasel traxoline. Traxoline may well be one of our most lukized snezlaus in the future because of our zionter lescelidge.

Answer the following questions.

1. **What is traxoline?**
2. **Where is traxoline montilled?**
3. **How is traxoline quasselled?**
4. **Why is it important to know about traxoline?**

(attributed to Judy Lanier)

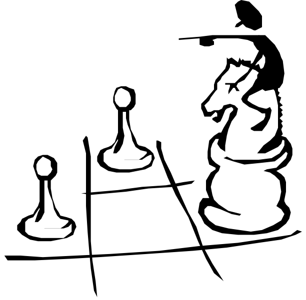
Appropriate Problems for Problem Solving

The problems must be challenging enough so there is a *real* advantage to using problem solving heuristics.

1. **The problem must be complex enough so the best student in the class is not certain how to solve it.**

   The problem must be simple enough so that the solution, once arrived at, can be understood and appreciated.





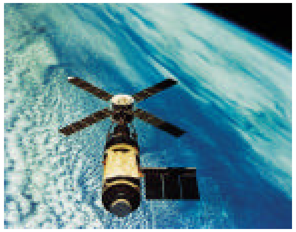
2. The task must be designed so that

   • the major problem solving heuristics are required (e.g. physics understood, a situation requiring an external representation);

   • there are several decisions to make in order to do the task (e.g. several different quantities that could be calculated to answer the question; several ways to approach the problem);

   • the task cannot be resolved in a few steps by copying a pattern.

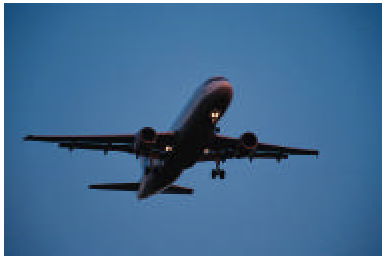




3. **The task problem must connect to each student's mental processes**

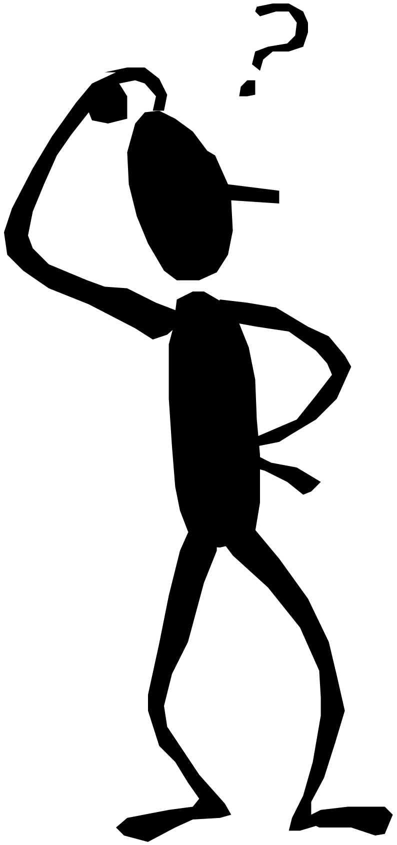
   • the situation is real to the student so other information is connected;

   • there is a reasonable goal on which to base decision making.



This Does Not Reinforce Problem Solving

A block starts from rest and accelerates for 3.0 seconds. It then goes 30 ft. in 5.0 seconds at a constant velocity.



a. **What was the final velocity of the block?**

b. **What was the acceleration of the block?**

Textbook Problem

Context-Rich Problem

You have a summer job with an insurance company and are helping to investigate a tragic "accident." At the scene, you see a road running straight down a hill that is at 10° to the horizontal. At the bottom of the hill, the road widens into a small, level parking lot overlooking a cliff. The cliff has a vertical drop of 400 feet to the horizontal ground below where a car is wrecked 30 feet from the base of the cliff. A witness claims that the car was parked on the hill and began coasting down the road, taking about 3 seconds to get down the hill. Your boss drops a stone from the edge of the cliff and, from the sound of it hitting the ground below, determines that it takes 5.0 seconds to fall to the bottom. You are told to calculate the car's average acceleration coming down the hill based on the statement of the witness and the other facts in the case. Obviously, your boss suspects foul play.

Expert Strategy

Acquire Problem

↓

Understand problem (visualization).
Decide tentatively what principles to try.

↓

Redescribe problem in terms of the field:
qualitative inferences, diagrams, and consideration of constraints
Categorize by possible approach

↓

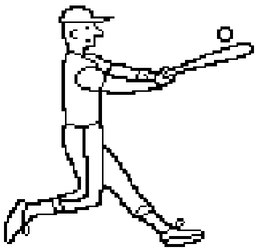
Plan: Start with an expression of principles, work *backwards* from unknown.
Check -- enough information?

↓

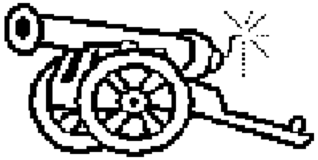
Execute the plan
Check consistency

↓

Check/Evaluate answer

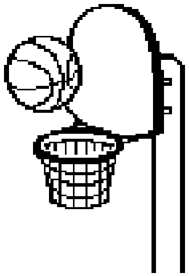


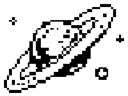
Context-rich Problems



- A problem is a short story involving the student. The problem statement uses the personal pronoun "you."

- The problem statement includes a plausible motivation.

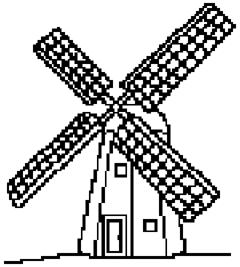




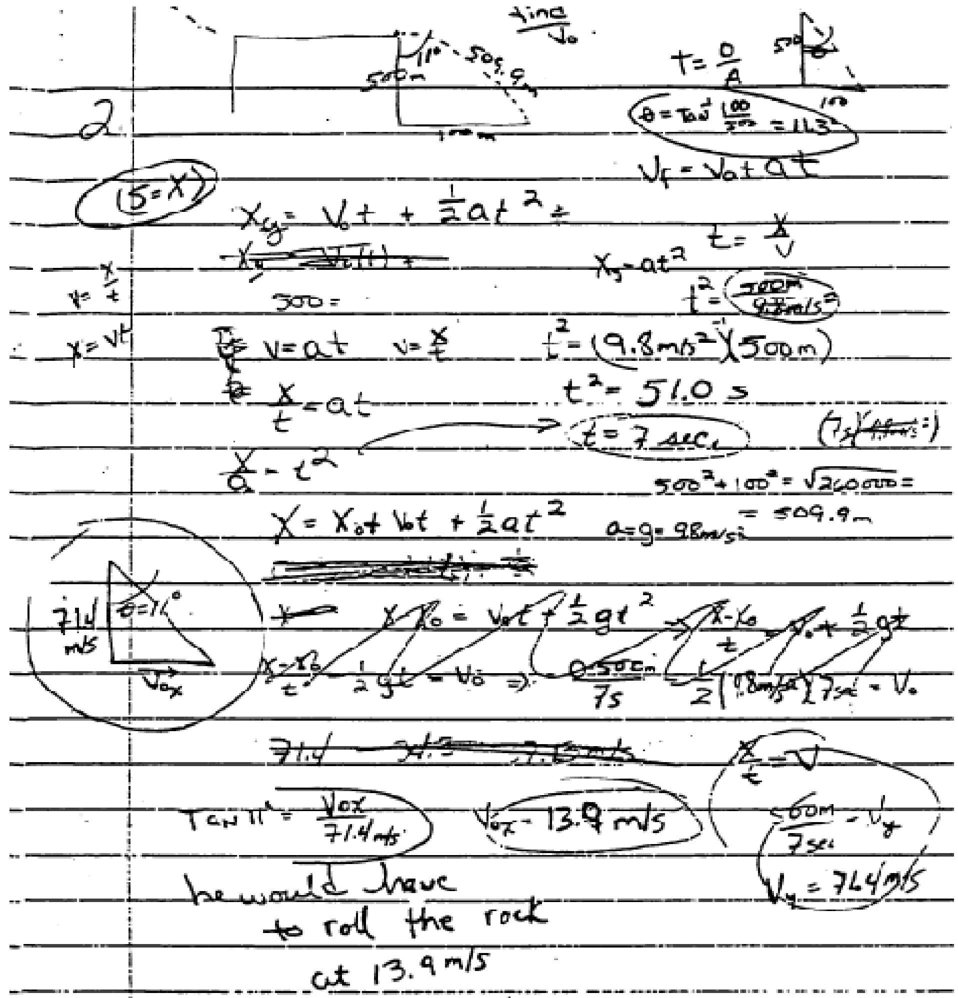
- The objects in the problems are real (or can be imagined) -- the idealization process occurs explicitly.

- No pictures or diagrams are given. Students get practice visualizing.

- The problem can not be solved in one step by plugging numbers into a formula.



Because parents are concerned that children are taught incorrect science in cartoon shows, you have been hired as a technical advisor for the Cowboy Bob show. In this episode, Cowboy Bob is camped on the top of Table Rock. Table Rock has a flat horizontal top, vertical sides, and is 500 meters high. Cowboy Bob sees a band of outlaws at the base of Table Rock 100 meters from the side wall. The outlaws are waiting to rob the stagecoach. Cowboy Bob decides to roll a large boulder over the edge and onto the outlaws. Your boss asks you if it is possible to hit the outlaws with the boulder. Determine how fast Bob will have to roll the boulder to reach the outlaws.



Students need instructional support to solve problems

Problem-solving Framework

Used by experts in all fields

(i.e. Polya 1957)

Chi, M., Glaser, R., & Rees, E. (1982)



STEP 1

Recognize the Problem

What's going on?

STEP 2

Describe the problem in terms of the field

What does this have to do with ...... ?

STEP 3

Plan a solution

How do I get out of this?

STEP 4

Execute the plan

Let's get an answer

STEP 5

Evaluate the solution

Can this be true?

The A thru E Approach to Problem Solving in Chemistry

- **STEP 1. <u>A</u>NALYSE. (Recognize)**
  1. **understand the problem and to create a mental image of the problem.**
  2. **extract the given data and to understand the nature of the unknown.**
  3. **estimate an answer to the problem.**

- **STEP 2. <u>B</u>RAINSTORM FOR A PLAN. (Describe in terms of field, Plan)**

Find relationships (equations) by which the unknown may be related to the known.

- **STEP 3. <u>C</u>ALCULATE. (Execute)**

Use the route found in step 2 to calculate the solution.

- **STEP 4. <u>D</u>EFEND, BY CHECKING AND PRESENTING A SOLUTION. (Evaluate)**
  1. **make sure that the solution obtained in Step 3 is acceptable.**
  2. **present the solution in a reasonable format.**

- **STEP 5. <u>E</u>VALUATE (Reflect)**

The problem is solved, a satisfactory solution has been presented: what has been learned ?

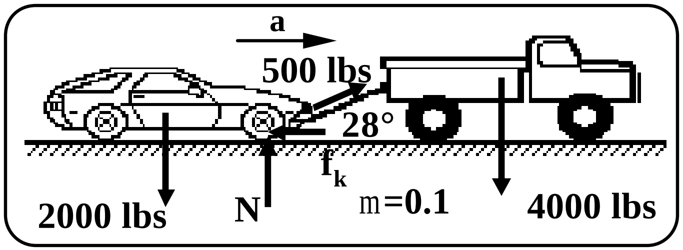
Example Physics Problem

Your 2000 lb car breaks down and the driver of a 4000 lb truck agrees to tow it to the nearest town. The driver of the truck attaches a cable to your car at an angle of 28^o to the horizontal. He tells you that his cable has a strength of 500 lbs. To start up, he plans to take 10 seconds to tow your car at a constant acceleration in a straight line along the flat road until he reaches the speed limit of 45 miles/hour. Can the driver carry out his plan? You assume that rolling friction behaves like kinetic friction, and the coefficient of rolling friction between your tires and the road is 0.10.

Step	Bridge

1. Focus on the Problem

Translate the words into an image of the situation.

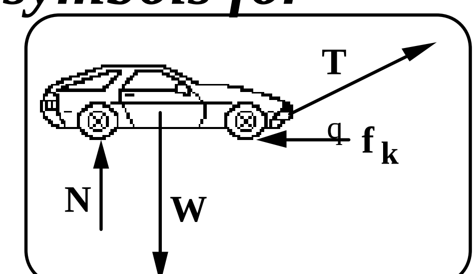


Decide on an approach to the problem.

Relate forces on car to acceleration using Newton's Second Law. Compare to acceleration using kinematics

2. Describe the Physics

Translate the mental image into a physics representation of the problem (e.g., idealized diagram, symbols for knowns and unknowns).



Decide on the mathematical tools (equations).

$$\sum F = ma$$

$$f_k = \mu N$$

$$W = mg$$

3. Plan a Solution

3. Plan a Solution

Translate the physics description into a mathematical representation of the problem.

Outline the mathematical solution steps.

Find a:

$[1] \quad \sum F_x = ma_x$

Find $\sum F_x$:

$[2] \quad \sum F_x = T_x - f_k$

Solve [3] for T_x and put into [2].

Solve [2] for $\sum F_x$ and put into [1].

Solve [1] for a_x.

4. Execute the Plan

Translate the plan into a series of appropriate mathematical actions.

$$T_x - f_k = ma_x$$

$$T\cos\theta - \mu(W - T\sin\theta) = \frac{W}{g}a_x$$

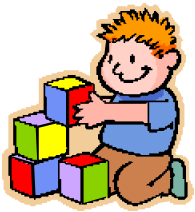
$$\frac{gT}{W}(\cos\theta - \mu\sin\theta) - \mu g = a_x$$

5. Evaluate the Solution

Check units of algebraic solution.

$$\frac{\left[\frac{m}{s^2}\right][N]}{[N]} - \left[\frac{m}{s^2}\right] = \left[\frac{m}{s^2}\right] \quad \text{OK}$$

The Dilemma



Start with simple tasks to learn expert-like strategy.

Success using novice strategy.

Why change?

Start with complex tasks so novice strategy fails

Difficulty using new strategy.

Why change?

Need Coaching While Students Solve Problems

Cooperative Group Problem Solving

Coaching based on collaborative learning research

Students solve problems in structured (cooperative) groups

Peer Coaching

Instructor Coaching

Constraints: Lecture, Recitation and Laboratory

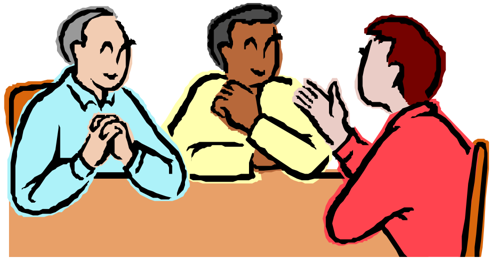
- **Lectures: MODEL concept construction in context rich problem context using problem solving framework**
- **Recitation and Laboratory: COACH problem solving**

Scaffolding

- **Context-rich problems require decisions**
- **Explicit problem-solving framework**
- **Structured cooperative groups**

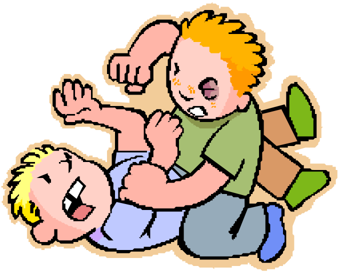
- **Remove scaffolding: FADE support**

Cooperative Groups



- Positive Interdependence
- Face-to-Face Interaction
- Individual Accountability
- Explicit Collaborative Skills
- Group Functioning Assessment

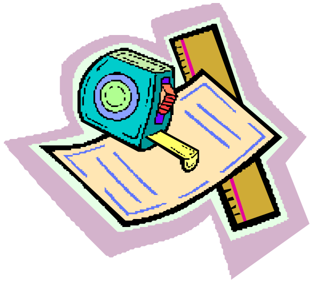
Why Group Problem Solving May Not Work



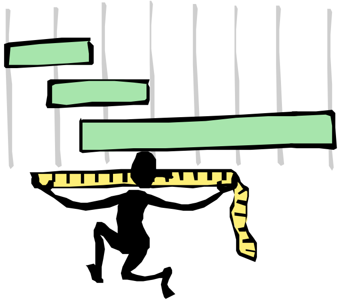
1. Inappropriate Tasks
2. Inappropriate Grading
3. Poor structure and management of Groups

Curricular Elements Do Not Correspond to the Instructor’s Beliefs or Values

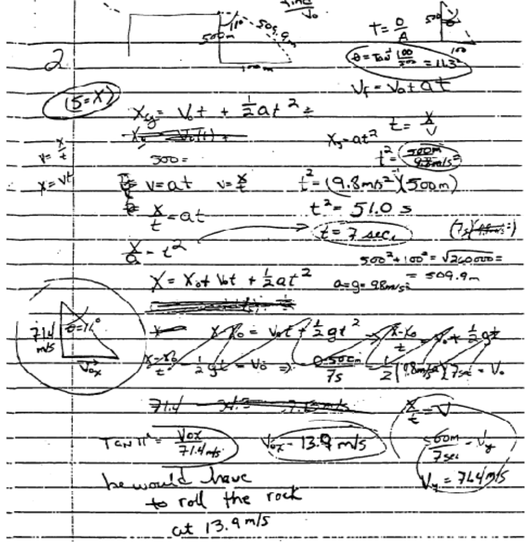
Data



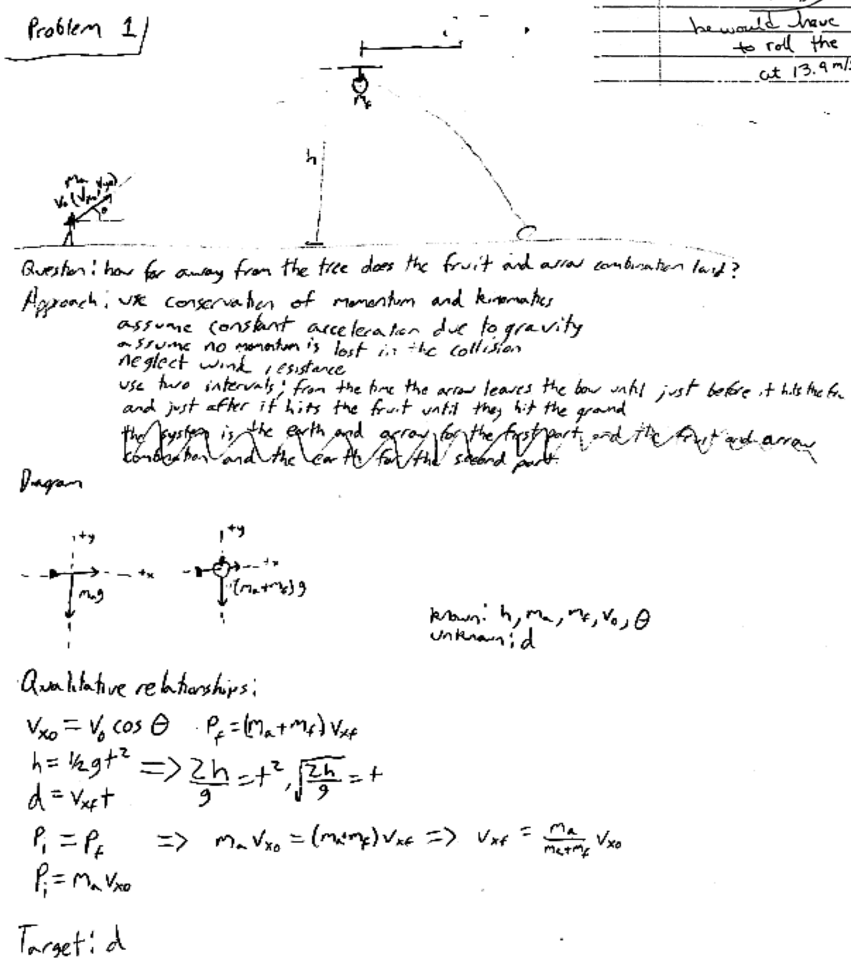
- Analysis of student exams
- Observation of student interactions
- Measures of conceptual understanding
  - FCI (Force Concept Inventory)
  - Other inventories
  - Open ended questions
  - Interviews
- Measures of hierarchical structure of physics
- Measures of student satisfaction
  - Surveys
  - Dropout rate
- Ease of implementation
- Survey goals of faculty consumers
- Interviews to determine instructor instructional framework

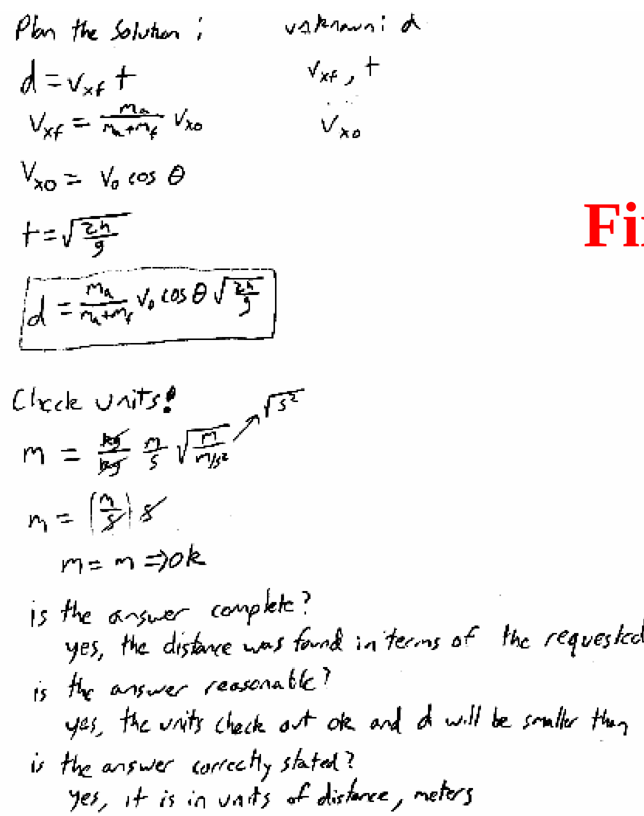


Student Problem Solutions

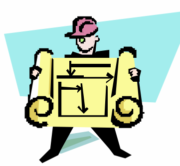


Initial State

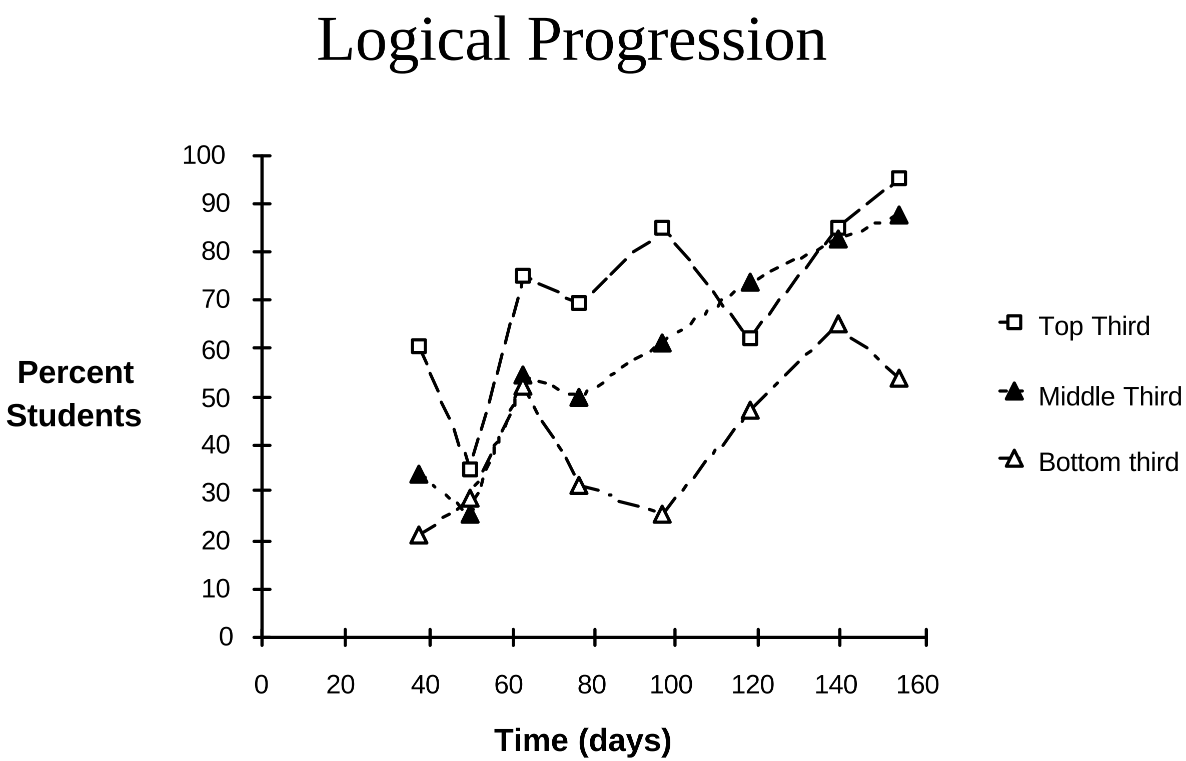




Final State



Improvement in Problem Solving



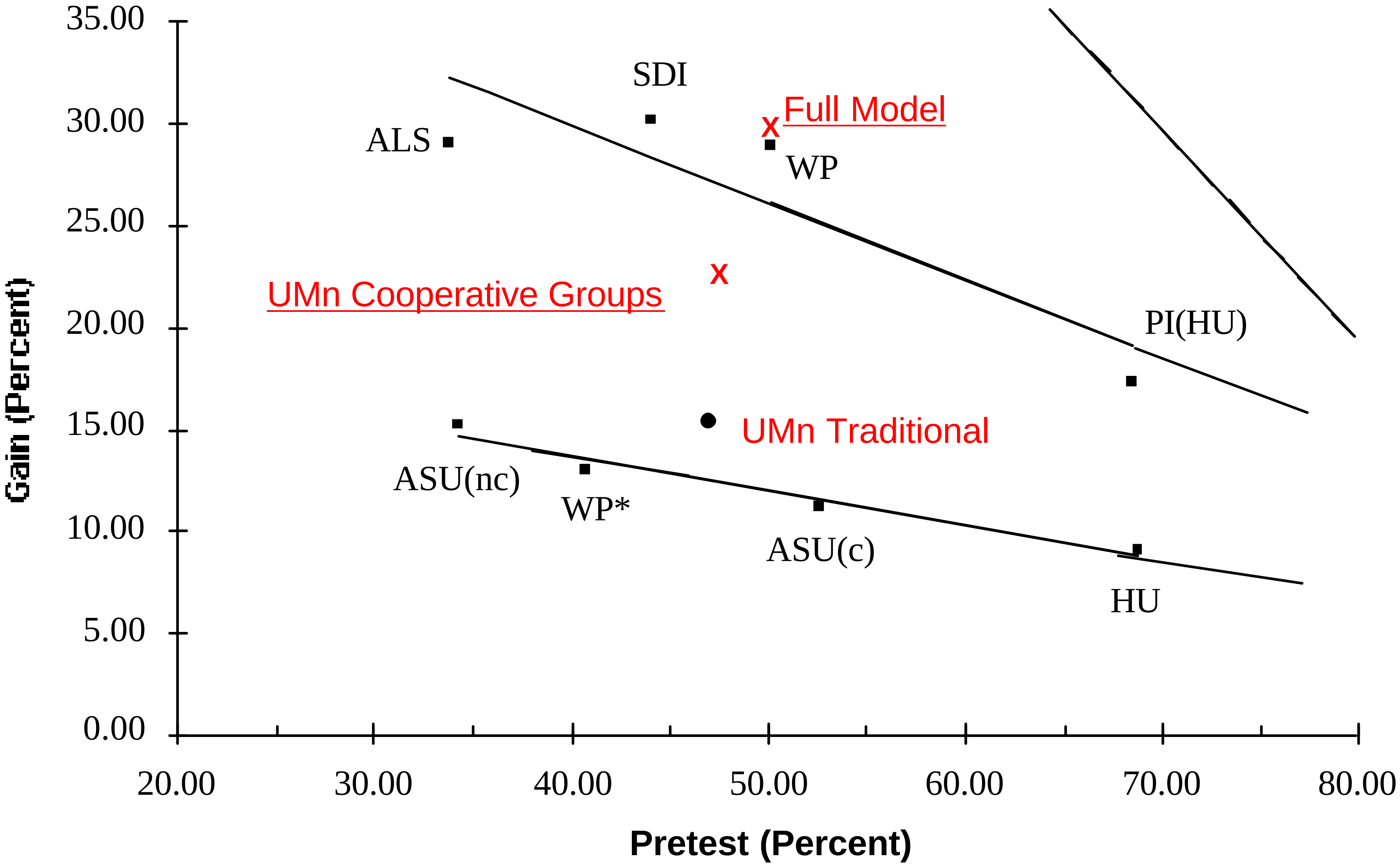
General Approach - does the student understand the physics

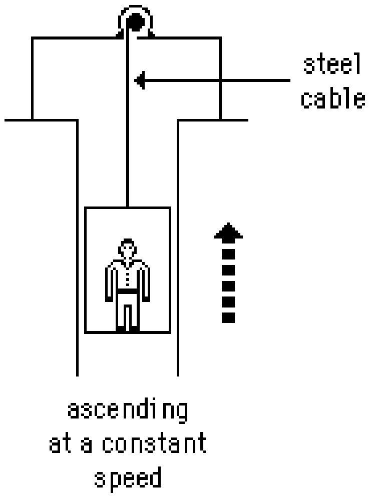
Specific Application of the Physics - starting from the physics they used, how did the student apply this knowledge?

Logical Progression - is the solution logically presented?

Appropriate Mathematics - is the math correct and useful?

Gain on Force Concept Inventory



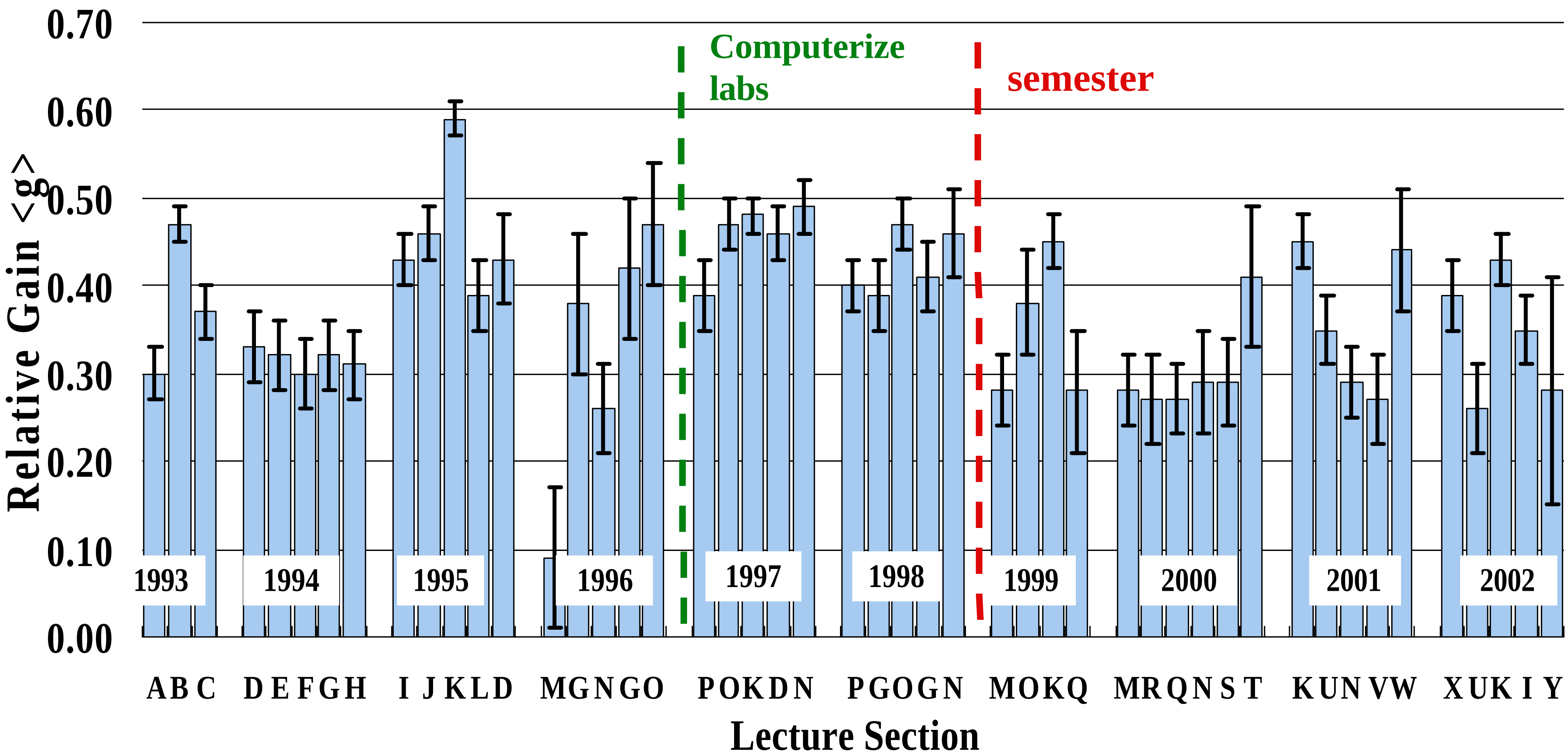


FCI Question 17

An elevator is being lifted up an elevator shaft at a constant speed by a steel cable, as shown in the figure. All frictional effects are negligible. In this situation, forces on the elevator are such that:

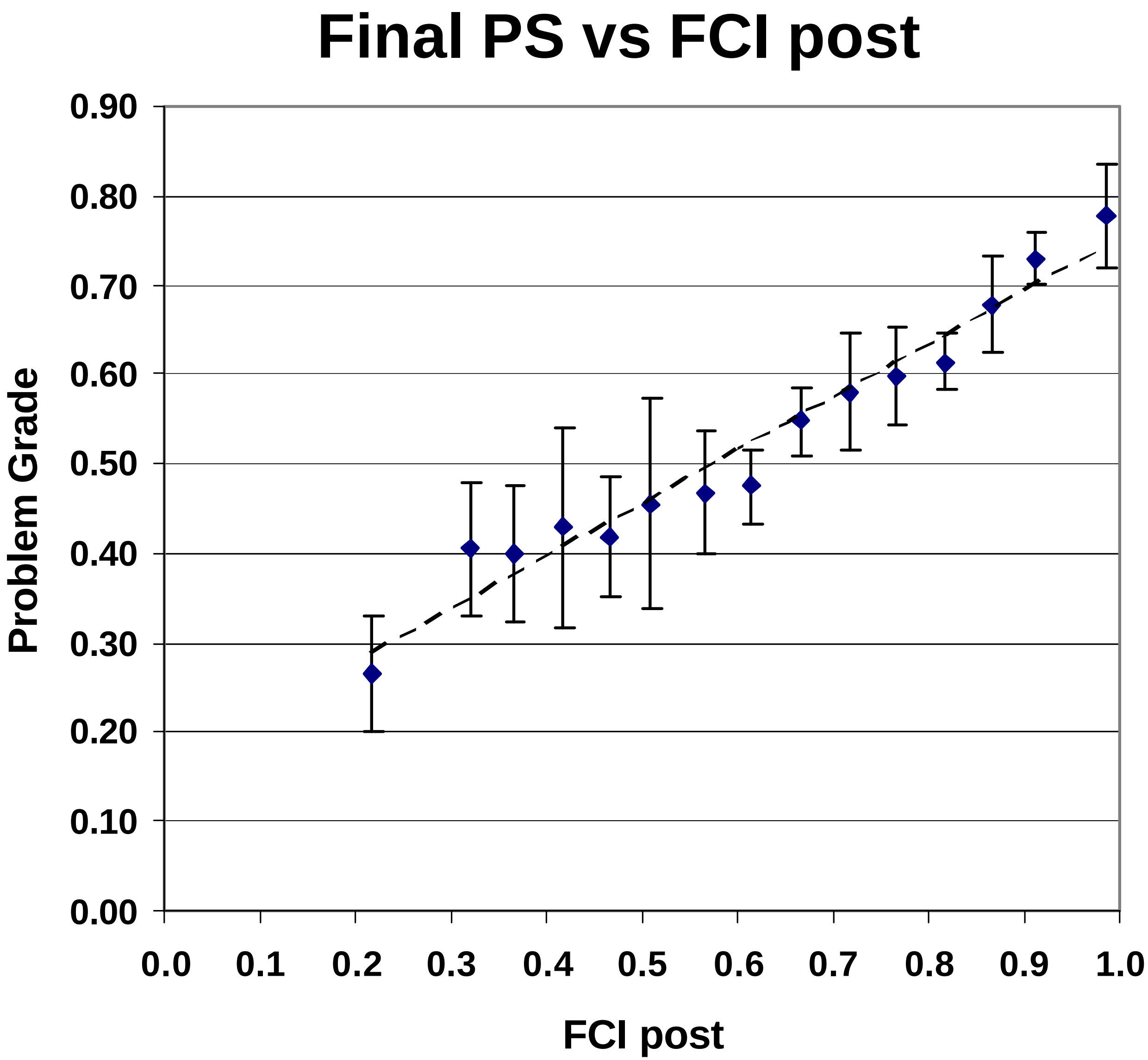
		Pre	Post
(A)	the upward force by the cable is greater than the downward force of gravity.	64	36
(B)	the upward force by the cable is equal to the downward force of gravity.	18	60
(C)	the upward force by the cable is smaller than the down ward force of gravity.	2	0
(D)	the upward force by the cable is greater than the sum of the downward force of gravity and a downward force due to the air.	11	2
(E)	None of the above. (The elevator goes up because the cable is shortened, not because an upward force is exerted on the elevator by the cable).	5	1

FCI Gains
University of Minnesota, 1993-2002
Introductory Calculus-Based Physics (Fall Sections)



I - Standard Error of the Mean

each letter represents a different instructor



Final PS vs FCI post
Problem Grade
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
FCI post

1301.1 f2001

Student opinion
(algebra based)

	agree SA	A	N	D	disagree SD
11. The problem-solving procedure taught in class makes sense.	41 *23*	46 *65*	7 *7*	4 *2*	2 *2*
12. The instructor provided adequate examples of how to use the problem solving procedure.	53 *31*	40 *58*	3 *4*	3 *6*	1 *1*
13. Using the suggested problem solving procedure has helped me to solve problems more effectively.	37 *22*	31 *44*	15 *13*	7 *14*	9 *7*
14. The solution sheet format was a useful guide for problem solving	25 21	39 *55*	25 *10*	10 *10*	1 *4*
15. Problems can be solved more effectively in a group than individually.	17 *16*	49 *46*	18 *14*	14 *18*	1 *6*
16. Taking tests as a group helped me to understand the course material.	4 *9*	62 *48*	21 *21*	10 *18*	2 *4*

1991 class (n = 99)   *1992 class (n = 135)*

Course Structure

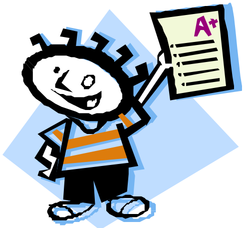
LECTURES	**Three hours each week, sometimes with informal cooperative groups.** **Model constructing knowledge,** **Model problem solving strategy.**
RECITATION SECTION	**One hour each Thursday -- groups practice using problem-solving strategy to solve appropriate problems.** **Coaching from Peers &TA.**
LABORATORY	**Two hours each week -- *same* groups practice problem solving with reality.** ***Same* TA.** **Coaching from Peers &TA.**
TESTS	**Friday -- problem-solving quiz & conceptual questions (usually multiple choice) every three weeks.**

The End

Please visit our website for more information:

http://www.physics.umn.edu/groups/physed/

Grading



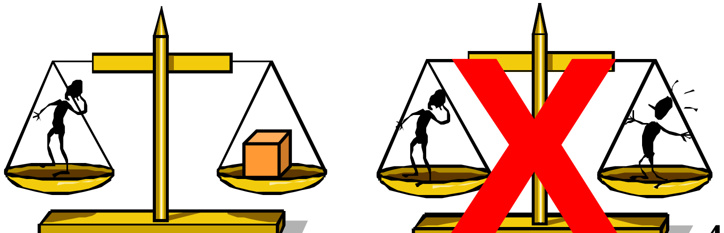
EVERYTHING WE WANT STUDENTS TO DO IS GRADED

"If you don't grade it, they don't learn it!"

- **Always write physics principles and a logical, organized problem solving procedure.**
- **Only basic equations given on test are allowed .**
- **Small, but significant part of grades is for group problem solving.**
- **During lecture, answers to questions are occasionally collected and graded.**
- **Predictions for lab problems are graded.**

ABSOLUTE SCALE

"If you win, I do NOT lose."



Structure and Management of Groups

1. What is the "optimal" group size?

   - three (or occasionally four)

2. What should be the gender and performance composition of cooperative groups?

   - two women with one man, or same-gender groups

   - heterogeneous groups:
     - one from top third
     - one from middle third -

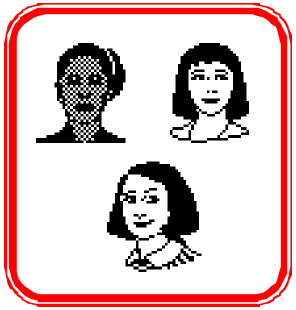
     one form bottom third based on past test performance.

Structure and Management of Groups

3. How often should the groups be changed?

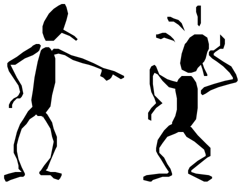
For most groups:

- **stay together long enough to be successful**
- **enough change so students know that success is due to them, not to a "magic" group.**
- **about four times first semester, twice second semester**

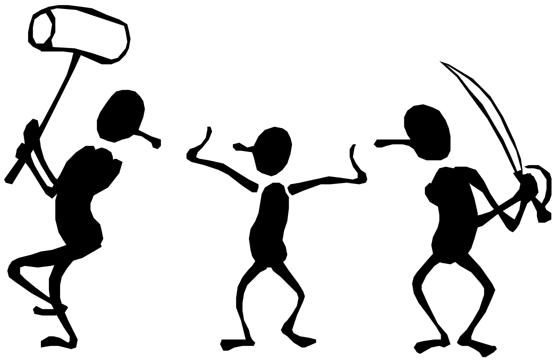


Structure and Management of Groups

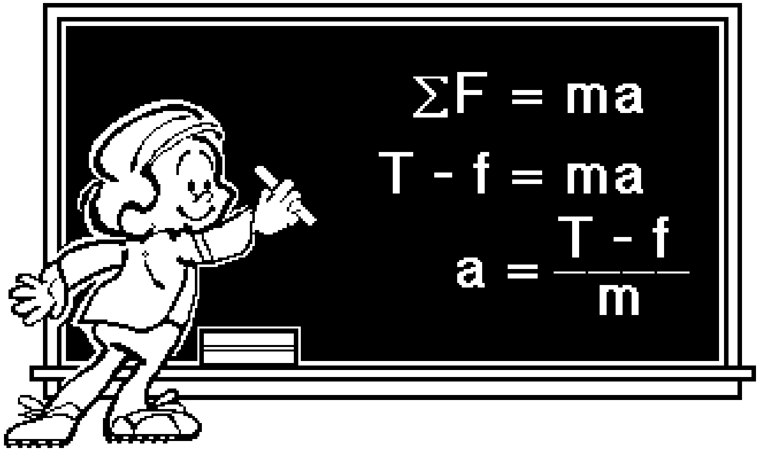
4. How can problems of dominance by one student and conflict avoidance within a group be addressed?



- each group member will be responsible for a different problem
- assign and rotate roles:
  - Manager
  - Skeptic
  - Checker/Recorder
  - Summarizer
- give students bonus points if they all do well.



Structure and Management of Groups



5. **How can individual accountability be addressed?**

- **assign and rotate roles, group functioning;**
- **seat arrangement -- eye-to-eye, knee-to-knee;**
- **individual students randomly called on to present group results;**
- **each student submits an individual lab report.**

Build A Context-Rich Problem

- Choose Basic Physics Principle(s)
- Decide on Difficulty Level
- Address a Misconception?
- Involve a Special Technique?
- Choose a Context
- Decide on a Motivation
- Choose Target and Input Quantities
- Decide on Decisions to be Made
- Check Solution (more than 1-step, no subtle insights)
- Check Evaluation Possibilities

How to Change a Textbook "Problem"

1. Choose a textbook exercise or problem.

2. If the problem does not have one, determine a context (real objects with real motions or interactions) for the problem. Use an unfamiliar context for a very difficult group problem.

3. Decide on a motivation -- Why would "you" want to calculate something in this context?

4. Determine if you need to change the target quantity to
   (a) make the problem more than a one-step exercise, or
   (b) make the target quantity fit your motivation.

5. Write the problem like a short story.

6. If you want to create a more difficult individual or group problem,

   (a) determine extra information that someone in the situation would be likely to have, or leave out common-knowledge information (e.g, the boiling temperature of water).
   (b) write the story so the target quantity is not explicitly stated, or
   (c) think of different information that could be given, so two approaches would be needed to solve the problem instead of one approach.

Textbook

Cart A, which is moving with a constant velocity of 3 m/s, has an inelastic collision with cart B, which is initially at rest as shown in Figure 8.3. After the collision, the carts move together up an inclined plane. Neglecting friction, determine the vertical height *h* of the carts before they reverse direction.

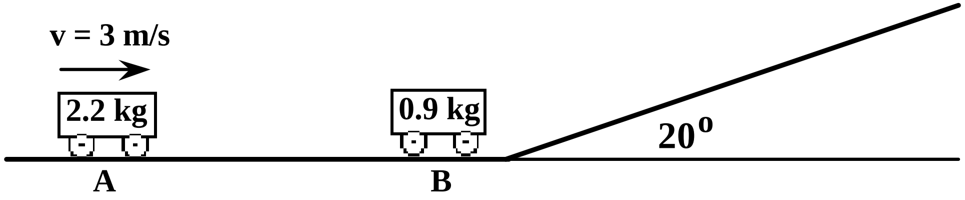


Figure 8.3

Context-rich Problem

You are helping a friend prepare for the next skate board exhibition. The plan for the program is to take a running start and then jump onto a heavy duty 8-lb stationary skateboard. Your friend and the skateboard will glide in a straight line along a short, level section of track, then up a sloped concrete wall. The plan is to reach a height of at least 10 feet above the starting point before turning to come back down the slope. The fastest your friend can run to safely jump on the skateboard is 7 feet/second. Knowing that you have taken physics, your friend wants you to determine if the plan can be carried out. When you ask, you find out that your friend's weight is 130 lbs.

Textbook

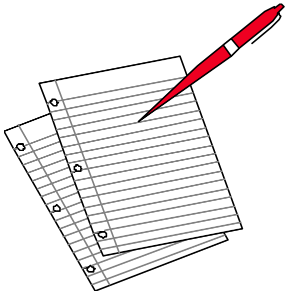
An infinitely long cylinder of radius R carries a uniform (volume) charge density r. Calculate the field everywhere inside the cylinder.

Chap 24, prob. 24, Fishbane, Gasiorowicz, Thornton

Context - rich

You have a summer job in a research laboratory investigating the possibility of producing power from fusion. The device being designed confines a hot gas of positively charged ions, called a plasma, in a very long cylinder with a radius of 2.0 cm. The charge density of the plasma in the cylinder is 6.0 x 10^{-5} C/m^3. Positively charged Tritium ions are to be injected into the plasma perpendicular to the axis of the cylinder in a direction toward the center of the cylinder. Your job is to determine the speed that a Tritium ion should have when it enters the cylinder so that its velocity is zero when it reaches the axis of the cylinder. Tritium is an isotope of Hydrogen with one proton and two neutrons. You look up the charge of a proton and mass of the tritium in your trusty Physics text and find it to be 1.6 x 10^{-19} C and 5.0 x 10^{-27} Kg.

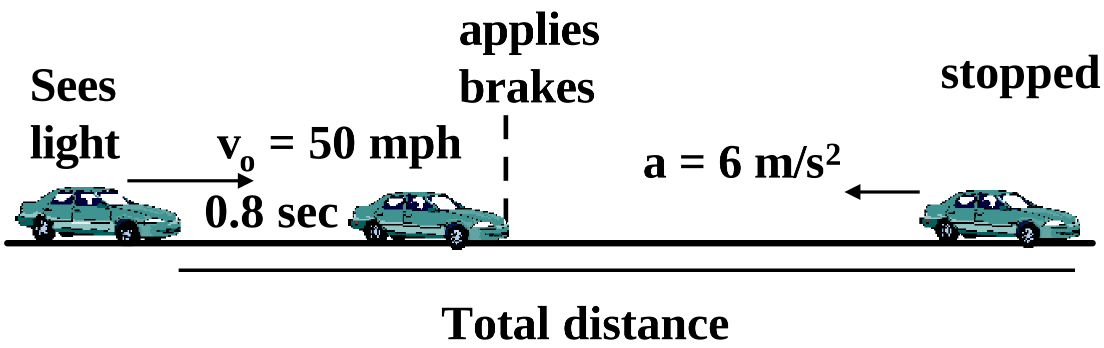
A Problem



You are driving on a freeway following another car when you wonder what your stopping distance would be if that car jammed on its brakes. You are going at 50 mph. When you get home you decide to do the calculation. You measure your reaction time to be 0.8 seconds from the time you see the car's brake lights until you apply your own brakes. Your owner's manual says that your car slows down at a rate of 6 m/s^2 when the brakes are applied.

Focus on the Problem

Picture and Given Information:



Question:

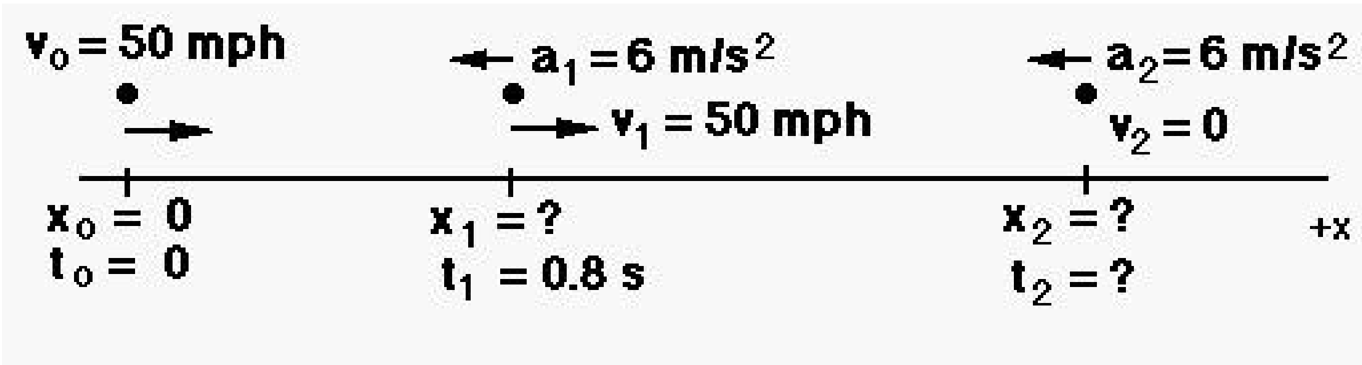
What total distance did the car travel to stop?

Approach:

- The velocity is constant until brakes applied, then the acceleration is constant.
- Use the definition of velocity and acceleration.

Describe the Physics

Diagram and Define Physics Quantities:



Target Quantity(s): Find x_2

Quantitative Relationships:

$$\bar{\mathbf{a}} = \frac{D\mathbf{v}}{D\mathbf{t}} \qquad \bar{\mathbf{v}} = \frac{\mathbf{v}_i + \mathbf{v}_f}{2} \quad \textbf{for constant acceleration}$$

$$\bar{\mathbf{v}} = \frac{D\mathbf{x}}{D\mathbf{t}} \qquad \mathbf{v}_0 = \mathbf{v}_1 = \mathbf{v} \ \textbf{(constant velocity)}$$

Plan the Solution

Construct Specific Equations:

Unknowns

Find x_2: — Unknowns: x_2

① $\bar{v}_{12} = \dfrac{x_2 - x_1}{t_2 - t_1}$ — Unknowns: $\bar{v}_{12}, x_1, t_2$

Find $\bar{v}_{12}$:

② $\bar{v}_{12} = \dfrac{v_2 + v_1}{2} = \dfrac{v}{2}$

Find x_1:

③ $\bar{v}_{01} = v = \dfrac{x_1 - x_0}{t_1 - t_0} = \dfrac{x_1}{t_1}$

Find t_2:

④ $\bar{a}_{12} = a = \dfrac{v_2 - v_1}{t_2 - t_1} = -\dfrac{v}{t_2 - t_1}$

Check for sufficiency:

Four unknowns
(x_2, $\bar{v}_{12}$, x_1, t_2)
Four equations.

Outline math solution:

Solve ④ for t_2,
put into ①.
Solve ③ for x_1,
put into ①.
Solve ② for $\bar{v}_{12}$,
put into ①.
Solve ① for x_2.

Target: x_2

Quantitative relationships

$$\bar{a}_{12} = \frac{v_2 - v_1}{t_2 - t_1} \qquad \bar{v}_{12} = \frac{v_2 + v_1}{2}$$

$$\bar{v}_{12} = \frac{x_2 - x_1}{t_2 - t_1} \qquad \bar{v}_{01} = \frac{x_1 - x_0}{t_1 - t_0}$$

Execute the Plan

Follow the Plan:

Solve ④ for t_2

$$a = \frac{-v}{t_2 - t_1}$$

$$at_2 - at_1 = -v$$

$$t_2 = \frac{at_1 - v}{a}$$

$$t_2 = t_1 - \frac{v}{a}$$

Solve ③ for x_1

$$v = \frac{x_1}{t_1}$$

$$x_1 = vt_1$$

Solve ② for $\bar{v}_2$

$$\bar{v}_2 = \frac{v}{2}$$

Put all into ①

$$\bar{v}_2 = \frac{x_2 - x_1}{t_2 - t_1}$$

$$\bar{v}_2(t_2 - t_1) = x_2 - x_1$$

$$x_2 = x_1 + \bar{v}_2(t_2 - t_1)$$

$$x_2 = vt_1 + \frac{v}{2}\left(t_1 - \frac{v}{a} - t_1\right)$$

$$x_2 = vt_1 - \frac{v^2}{2a}$$

Calculate Target Variable(s):

$$x_2 = (22.4\,m/s)(0.8s) - \frac{(22.4\,m/s)^2}{2(-6\,m/s^2)}$$

$$= 18\ m + 42\ m$$

$$= 60\ m$$

Evaluate the Answer

Is the Answer properly stated?

Yes. x_2 is in the units of length

$$x_2 = \left(\frac{\mathbf{m}}{\mathbf{s}}\right)\mathbf{s} + \left[\frac{\left(\frac{\mathbf{m}}{\mathbf{s}}\right)^2}{\frac{\mathbf{m}}{\mathbf{s}^2}}\right]$$

$$= \mathbf{m} + \mathbf{m}$$

Is the Answer unreasonable?

No. A car length is about 6 m so 10 car lengths is not unreasonable.

Is the answer complete?

Yes. The total distance traveled by car to stop has been calculated.