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Developing a Useful Instrument to Assess Student Problem Solving

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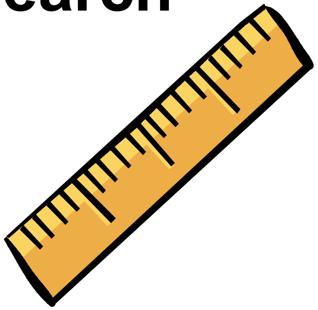
Problem Solving Measure

- Problem solving is one of the primary teaching goals, teaching tools, and evaluation techniques of physics courses.

- Issues:
  - There is no standard way to evaluate problem solving that is valid, reliable, and easy to use.
  - A single numerical score gives an inadequate description of a student's skill at solving problems.

- A more detailed and meaningful measure of problem solving would be useful for both research and instruction.

- How do we measure a complex skill such as problem solving?



What is *problem solving*?

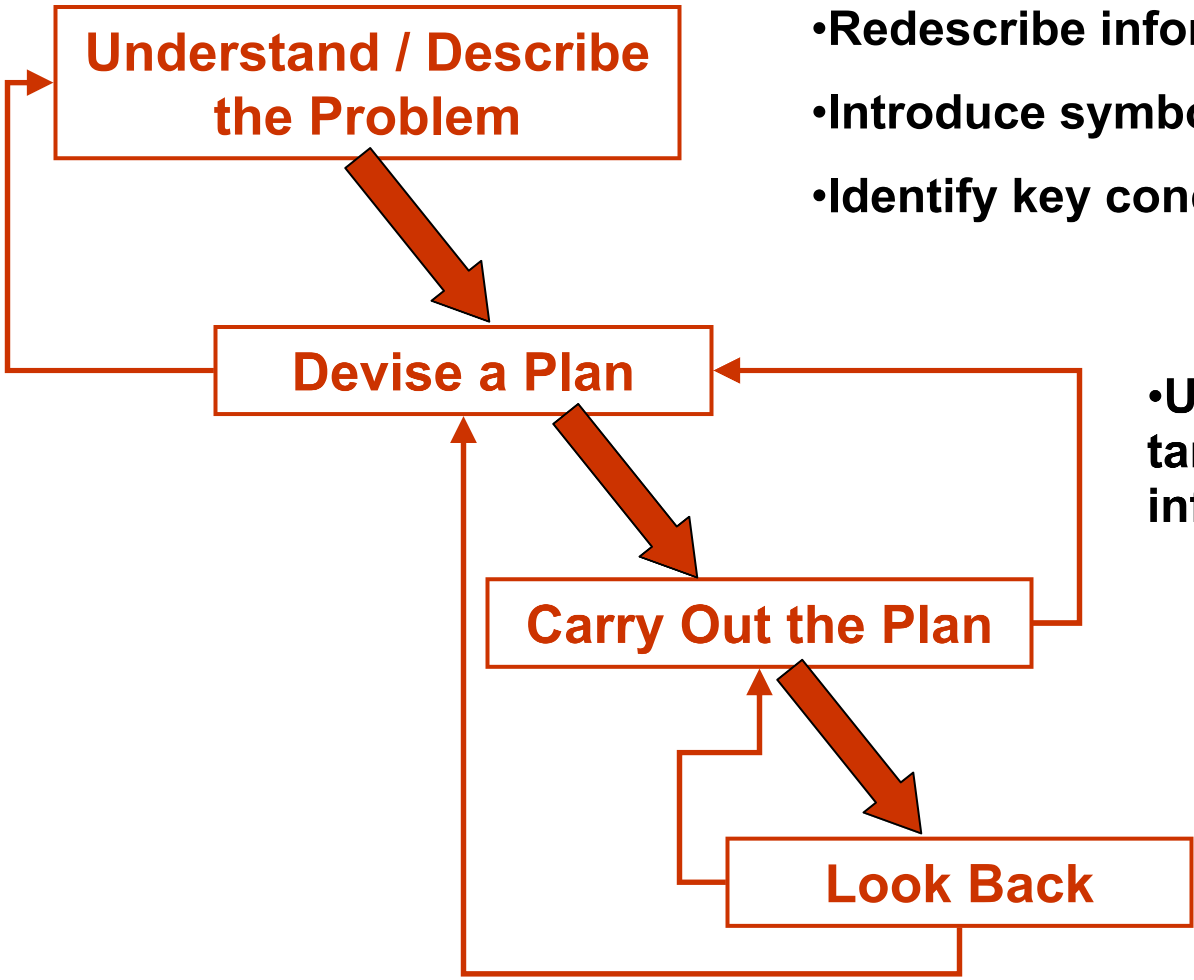
- "**Problem solving** is the process of moving toward a goal when the path to that goal is uncertain" (Martinez, 1998, p. 605)

- What is a ***problem*** for one person might not be a problem for another person.

- Problem solving involves decision-making.
  - If the steps to reach a solution are immediately known, this is an ***exercise*** for the solver.

Martinez, M. E. (1998). What is Problem Solving? *Phi Delta Kappan*, *79*, 605-609.
Schoenfeld, A.H. (1985). *Mathematical problem solving*. Orlando, FL: Academic Press, Inc.

Problem Solving Process



- Redescribe information from problem
- Introduce symbolic notation
- Identify key concepts

- Use concepts to relate target to known information

- appropriate math procedures

- check answer

Pólya, G. (1957). *How to solve it* (2nd ed.). Princeton, NJ: Princeton University Press.

Reif, F. & Heller, J.I. (1982). Knowledge structure and problem solving in physics. *Educational Psychologist*, *17*(2), 102-127.

Problem solver characteristics

Inexperienced solvers:

- Knowledge disconnected
- Little representation (jump to equations)
- Inefficient approaches (formula-seeking & solution pattern matching)
- Early number crunching
- Do not evaluate solution

Experienced solvers:

- Hierarchical knowledge organization or *chunks*
- Low-detail overview of the problem before equations
  - *qualitative analysis*
- Principle-based approaches
- Solve in symbols first
- Evaluate their solution

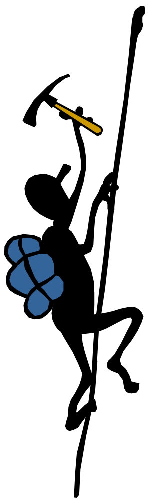
Chi, M. T., Feltovich, P. J., & Glaser, R. (1980). Categorization and representation of physics problems by experts and novices. *Cognitive Science*, *5*, 121-152.

Larkin, J., McDermott, J., Simon, D.P., & Simon, H.A. (1980). Expert and novice performance in solving physics problems. *Science*, 208(4450), 1335-1342.

Project Description

***Goal*:**

- **Use this research literature and data (problem solutions) to design a robust instrument to evaluate written solutions to physics problems**
  - **Establish criteria for appropriate use & training**



- **The instrument should be *general***
  - **not specific to instructor practices or technique**
  - **applicable to a variety of problem types and topics**
- **Must consider issues of *reliability* & *validity*:**
  - ***reliability* –** ***stability of scores across different raters***
  - ***validity* –** ***the instrument measures what it claims to measure (score interpretation is supported by empirical evidence and theoretical backing)***

Instrument at a glance (Rubric)

SCORE

CATEGORY:	5	4	3	2	1	0	NA(P)	NA(S)
Useful Description								
Physics Approach								
Specific Application								
Math Procedures								
Logical Progression								

Want

- ***Minimum*** ***number of categories that include relevant aspects of problem solving***
- ***Minimum*** ***number of scores that give enough information to improve instruction***
- ***Minimum*** ***training to use***

Rubric Scores (in general)

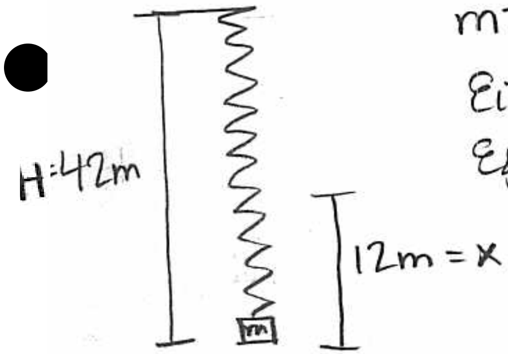
5	4	3	2
Complete & appropriate	Minor omissions or errors	Parts missing and/or contain errors	Most missing and/or contain errors

1	0	NA Problem	NA Solver
All inappro-priate	No evidence / missing	Not necessary for this problem	Not necessary for this solver

Sample Problem

- To raise money for a University scholarship fund, the new IT dean has volunteered to bungee jump from a crane if contributions can be found for 10 scholarships. To add some interest, the jump will be made from 42 m above a pool of water. A 30m bungee cord would be attached to the dean. First you must convince the dean that your plan is safe for a person of his mass, 70kg. Your plan has the dean stepping off a platform and being in free fall for 30 m before the cord begins to stretch.

- a) Determine the spring constant of the bungee cord so that it stretches only 12m, which will just keep the dean out of the water. (Assume that the dean is a point-like object).
- b) Find the dean's speed 7m above the water.



m = 70kg

$\mathcal{E}_i = mgH$

$\mathcal{E}_f = \frac{1}{2}kx^2$

The velocity = 0 when the dean is on the platform & when he is at the bottom of the jump. so KE = 0

$\mathcal{E}_i = \mathcal{E}_f$ by cons. of $\mathcal{E}$

$mgH = \frac{1}{2}kx^2$

$$k = \frac{2mgH}{x^2} = \frac{2(70kg)(9.8m/s^2)(42m)}{(12m)^2}$$

a) $\boxed{k = 400.2\ N/m}$

At 7m above water,

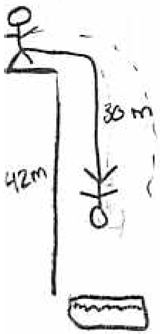
$\mathcal{E} = mgh + \frac{1}{2}kx_2^2 + \frac{1}{2}mv^2$ this is equal to mgH, where $x_2 = 12 - 7m = 5m$

$\frac{1}{2}mv^2 = mgH - mgh - \frac{1}{2}kx_2^2$

$$v = \sqrt{\frac{2mg(H-h) - \frac{1}{2}kx_2^2}{m}} = \sqrt{\frac{2(70kg)(9.8m/s^2)(42m-7m) - \frac{1}{2}(400N/m)(5m)^2}{70kg}}$$

$\boxed{v = 16.57\ m/s}$

USEFUL DESCRIPTION
PHYSICS APPROACH
SPECIFIC APPLICATION
MATH PROCEDURES
LOGICAL PROGRESSION



a. $x = 12\text{ m}$, $F = ma$, $a = -9.8\text{ m/s}^2$

$F = -kx$

$ma = -kx$

$(70\text{ kg})(-9.8\text{ m/s}^2) = -k(12\text{ m})$

$$\boxed{k = 57.167\ \frac{\text{N}}{\text{m}}}$$

USEFUL DESCRIPTION
PHYSICS APPROACH
SPECIFIC APPLICATION
MATH PROCEDURES
LOGICAL PROGRESSION

b. Use conservation of Energy

$PE_{top} = KE_{bottom} + PE_{bottom}$

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$ **$+ mgh_f$**

* The dean will have PE a[t] the top of his jump and the[n] KE and PE at the bottom b[e] the bungee will shoot him b[ack]

∴ x equals 12 m − 7 m = 5 m the spring will be stretched because that is the distance

$(70\text{ kg})(57.167)(42\text{ m}) = \frac{1}{2}(70\text{ kg})(v^2) + \frac{1}{2}(57.167)(5\text{ m})^2$

$$v^2 = \frac{(70\text{ kg})(57.167)(42) - \frac{1}{2}(57.167)(5)^2}{\frac{1}{2}(70\text{ kg})}$$

$$\boxed{v = 69.15\text{ m/s}}$$

$F_T = kx$ $\qquad$ $F_T = F_g$

$F_{g\,Dean} = mg$

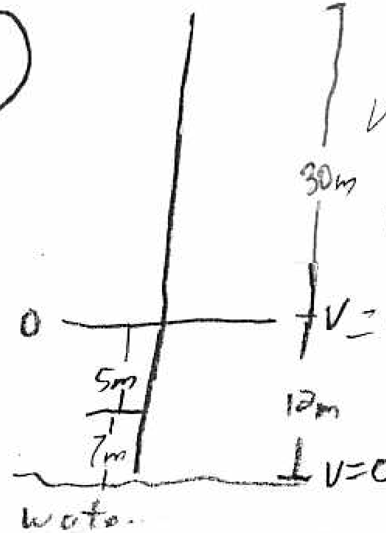
$kx = mg$

$k\,12 = 70g$

$k = \dfrac{70g}{12m}$

a) $\boxed{k = 57.23 \frac{N}{m}}$

b)



$v = \sqrt{2gy}$

$v = \sqrt{60g}$

$v = 24.26 \frac{m}{s}$

$v_f^2 = v_0^2 + 2ay$

?

$0 = 588.6 + 24a$

$a = -24.5 \frac{m}{s^2}$

$v_f^2 = 588.6 - 245.25$

$v_f = 18.53 \frac{m}{s}$

7m above the water

USEFUL DESCRIPTION

PHYSICS APPROACH

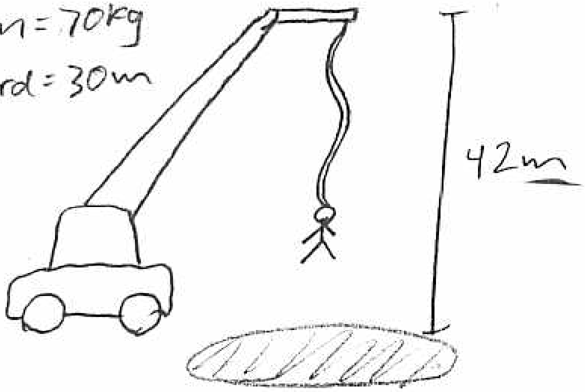
SPECIFIC APPLICATION

MATH PROCEDURES

LOGICAL PROGRESSION

$m = 70\,kg$

cord $= 30\,m$



$\boxed{F = -kx}$

a) $ma = -kx$

$(70\,kg)(9.8\,m/s^2) = k(-12\,m)$

$k = \frac{ma}{x} = \boxed{57.2\ N/m}$

b) $v^2 = v_0^2 + 2a(\Delta x)$

?

$v_0 = \sqrt{mgh}$ (where $h = 30\,m$)

$\Delta x = 5\,m$

$a = \frac{-\sqrt{mgh}}{12\,m}$

?

$V = \sqrt{\sqrt{mgh} + 2\left(\frac{-\sqrt{mgh}}{12\,m}\right)(5)}$

$V = \sqrt{143 + 2(-12)(5)} = \boxed{16\,m/s}$

USEFUL DESCRIPTION

PHYSICS APPROACH

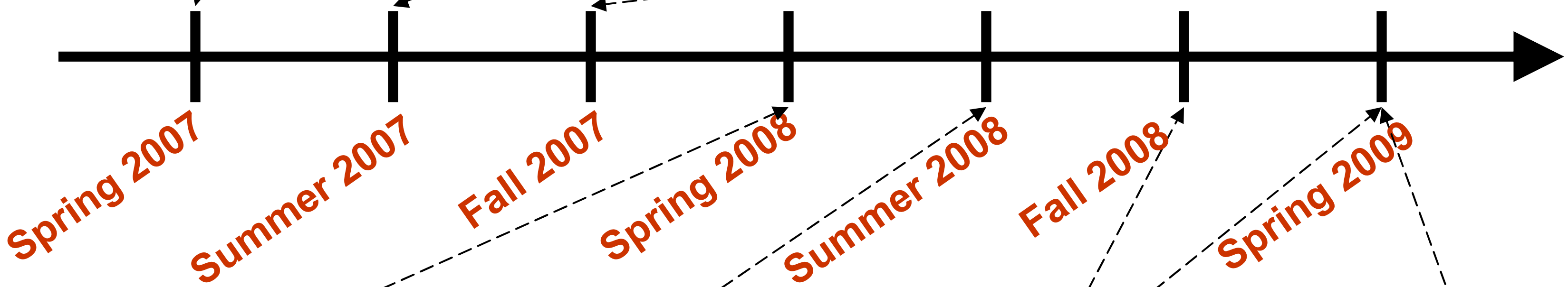
SPECIFIC APPLICATION

MATH PROCEDURES

LOGICAL PROGRESSION

Developing the Rubric

1. Draft instrument based on literature & archived exam data
2. Preliminary tests with two raters (consistency of scores)
3. Pilot instrument with graduate students (brief training)



4. Analyze pilot data (written feedback & scores)
5. Revise rubric and training materials
6. Re-pilot with graduate students &/or faculty; collect & score exams
7. Final data analysis & reporting

What have we learned?

- **Preliminary testing (two raters)**
  - **Distinguishes instructor & student solutions**
  - **Score agreement between two raters – good**

- **Pilot testing (8 Graduate Students)**
  - **confusion about NA scores (want more examples)**
  - **influenced by traditional grading experience**
    - ***Unwilling to score math & logic if physics incorrect***
  - **difficulty distinguishing approach & application**
  - **multi-part problems more difficult to score**
  - **score agreement improved slightly with training**

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Work in Progress!

Materials on the Web:

http://groups.physics.umn.edu/physed
(Click on Jennifer Docktor)

docktor@physics.umn.edu

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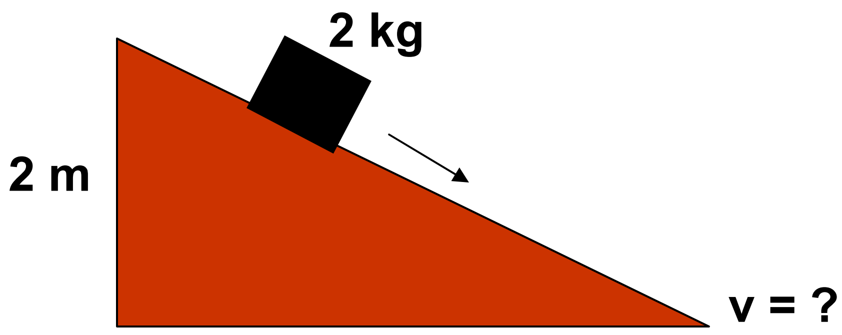
ADDITIONAL SLIDES

Example of NA (Problem)

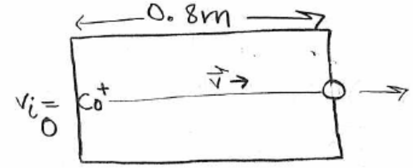


***Useful Description*:**
visual & symbolic representation given

A block of mass *m* = 2 kg slides down a frictionless ramp of height *h* = 2 m. Use conservation of energy to determine the speed of the block at the bottom of the ramp.

***Physics Approach*:**
physics concept or principle stated in problem

Range of detail in solutions



$s = 0.8$ m
$v_f = 8 \times 10^4$ m/s
$m_{co} = 28$ g/mol

$\xrightarrow{F_e}$ $qE = F_e = ma$

$E = \frac{ma}{q}$ $v_f^2 = \cancel{v_i^2}^{\,0} + 2a\Delta x$

$a = \frac{v_f^2}{2\Delta x}$

$E = \frac{m v_f^2}{q\, 2\Delta x} = \frac{[kg]\cdot[m]^{\cancel{2}}}{[s]^2 [C] [\cancel{m}]} = \frac{N}{C}$ $\frac{V}{m} = \frac{N}{C}$ $N = \frac{kg\, m}{s^2}$

units ok ✓

$\boxed{E = 1162 \frac{N}{C}}$

Question: Calculate the direction and magnitude of the electric field needed so that Co^+ ions created at rest at one end will have a speed of 8×10^4 m/s when they exit the other side

Approach: Use conservation of energy

system: Co^+ particle

intial time: right as Co^+ enters the box

final time: right as Co^+ leaves the box

$E_i = 0$ $v_i = 0$

$E_f = \frac{1}{2}mv^2$ E_{in} = electric potential energy, $E_{out} = 0$

Electric potential energy $= \Delta V q = -q\int \vec{E}\cdot d\vec{s}$

because the electric field is constant

$PE_e = -qE\int ds$

$PE_e = -qEs$ ← just want magnitude so can leave negative sign off

$E_f - E_i = E_{in} - E_{out}$ check units

$\frac{1}{2}mv^2 = qEs$ $\frac{1}{2}mv^2 \rightarrow$ energy units

$E = \frac{\frac{1}{2}mv^2}{qs}$ $\frac{V}{m}\cdot m, C = V\cdot C$ = energy units

units ok ✓

$|E| = \frac{\frac{1}{2}mv^2}{qs} =$ $J = N\cdot m$

$CO - e^- = CO^+$

$.028\,kg - 9.11 \times 10^{-31}\,kg = 0.028\,kg$

$|E| = \frac{\frac{1}{2}(0.028\,kg)(8 \times 10^4\,m/s)^2}{(1.602 \times 10^{-19}\,C)(0.8\,m)}$

$\boxed{|E| = 6.99 \times 10^{26}\ N/C}$

***Useful Description*: unnecessary for this solver NA(S)**

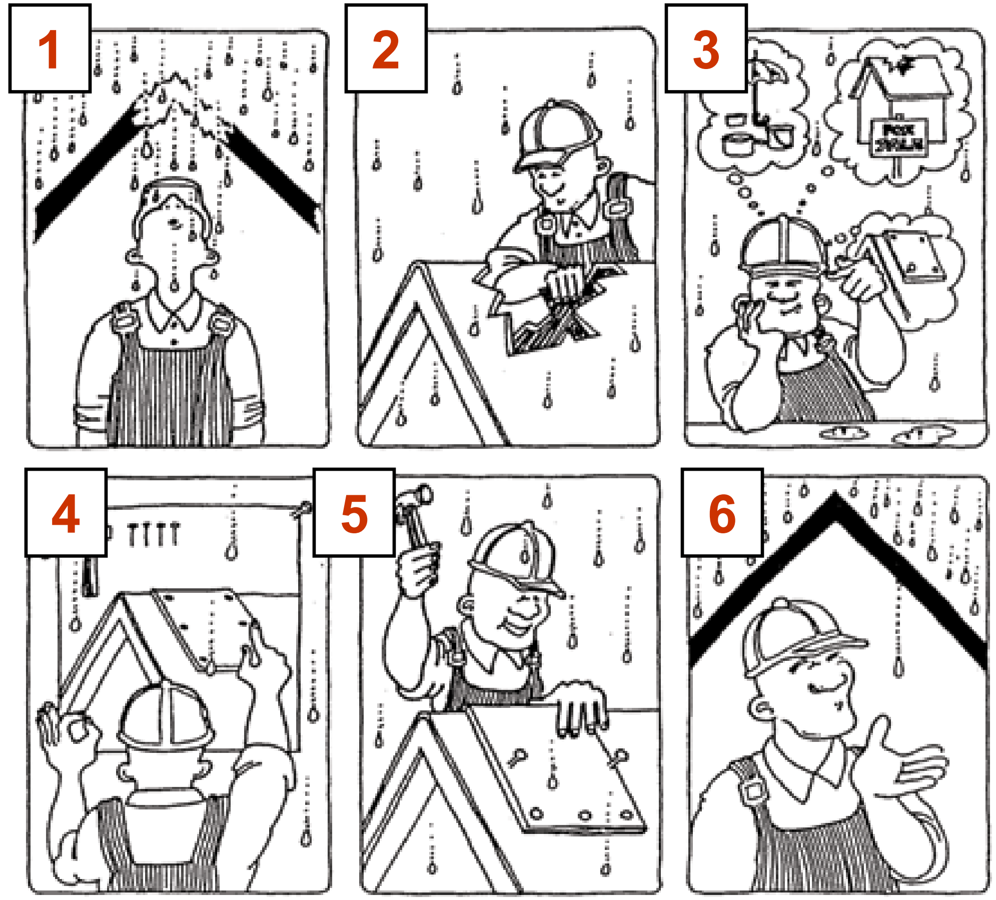
Rubric Categories (based on research literature)

- **Useful Description**
  - organize information from the problem statement symbolically, visually, and/or in writing.
- **Physics Approach**
  - select appropriate physics concepts and principles to use
- **Specific Application of Physics**
  - apply physics approach to the specific conditions in problem
- **Mathematical Procedures**
  - follow appropriate & correct math rules/procedures
- **Logical Progression**
  - (overall) solution progresses logically; it is coherent, focused toward a goal, and consistent

Note: 4 of the 5 categories are qualitative

Problem Solving Process

1. Identify & define the problem
2. Analyze the situation
3. Generate possible solutions/approaches
4. Select approach & devise a plan
5. Carry out the plan
6. Evaluate the solution



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